PPP Puzzle in a Heterogenous Sticky Price Model with Variable Markups

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ABSTRACT

In this paper, we study the real exchange rate (RER) dynamics in a multisector, sticky price model with local currency pricing, heterogeneity in frequency of price adjustment and variable markups. More specifically, we focus on producing the RER persistence seen in data for levels of nominal aggregate demand persistence matching the "nominal" exchange rate dynamics, unlike earlier studies that rely on high degrees of aggregate demand persistence. We find that heterogeneity in price stickiness alone amplifies the aggregate RER persistence considerably and brings it to the levels consistent with data. Despite this improvement, the model with constant markups still relies heavily on existence of high degrees of aggregate persistence. Introducing variable markups improves the aggregate RER persistence further and closes some of the gap between theory and data. Hence, the model with variable markups is superior to the constant markup model in explaining RER dynamics since it relies less on high degrees of nominal aggregate demand persistence. Yet, somewhat high (even though lower than before) degree of aggregate persistence is still needed to explain RER dynamics seen in data. Thus, consistent with the literature, we reach to the conclusion that the contribution of variable markups remains relatively modest and other sources of real rigidities should be sought.

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1 INTRODUCTION

Even though the term itself is relatively new, *Purchasing Power Parity (PPP)* has been studied long through the economic history. In its most simple and rigid form, PPP states that the nominal exchange rate between currencies of two countries should be equal to their aggregate price level ratios. Hence, PPP predicts that one unit of currency will have the same purchasing power in both countries. As a consequence, the real exchange rate (RER) between two countries should be constant at 1.[2] A less rigid form of PPP, on the other hand, asserts that RER should be constant, but not necessarily at 1. Neither of these versions of PPP are consistent with empirical findings. In one of the earlier studies, Mussa (1986) shows that both the nominal and real exchange rates are highly volatile compared to the relative price level and highly persistent. Rogoff (1996) provides a literature review on empirical studies and comes up with the conclusion that the half life of deviations from PPP ranges from 3 years to 5 years. This discrepancy between the theory and data is termed as *PPP puzzle* in the literature.

A large bulk of the literature on exchange rate determination is devoted to reconciling the theory with the well-documented volatility and persistence of the RER as seen in data.[3] The volatility side of the PPP puzzle is relatively easier to solve since models with monetary shocks and plausible degree of nominal rigidities are known to close the gap between data and theory in terms of generating high RER volatility. However, these models fail to replicate the large persistence seen in data unless the degree of nominal rigidities are unrealistically high.

In a recent study, Carvalho and Nechio (2011) introduce heterogeneity in frequency of price adjustment matching US data, and pricing to market into an otherwise standard multisector, two-country, sticky price model. They show that the heterogeneous multisector model generates both more volatile and persistent aggregate RER dynamics in response to nominal shocks compared to a one sector counterfactual model. Indeed, their heterogeneous multisector model produces a half life of 39 months, which falls within the range provided by Rogoff (1996). However, the success of the model of Carvalho and Nechio (2011) in generating realistic measures of RER dynamics relies on having a high degree of nominal aggregate demand persistence, which is not consistent with nominal exchange rate dynamics found in Imbs, Mumtaz, Ravn and Rey (2005). When nominal aggregate demand persistence is set to a level consistent with nominal exchange rate dynamics, the heterogeneous multisector model fails to generate realistic levels of aggregate RER persistence, even though it still does remarkably better than the counterfactual one sector economy. Carvalho and

[2]RER is defined as: $e\frac{P^*}{P}$, where e is nominal exchange rate, P^* and P are aggregate price levels in foreign and home countries respectively.

[3]See Rogoff (1996) and Taylor and Taylor (2004) for a review of the literature.

Nechio (2011) attribute this failure of their model to non-existence of real rigidities, other than the decreasing returns to scale (DRS) production technology. To investigate the likely effects of incorporating real rigidities into their model, they set elasticities of substitution across varieties and sectors at unrealistically high levels and predict that introducing real rigidities into their model would enable the model to explain RER dynamics even at nominal aggregate demand persistence levels consistent with nominal exchange rate dynamics.

Our aim in this paper is to test the prediction of Carvalho and Nechio (2011) by introducing real rigidities in the form of variable markups into their baseline model. More specifically, we are aiming at generating realistic aggregate RER persistence at a nominal aggregate demand persistence level consistent with the dynamics of nominal exchange rates, while keeping elasticities of substitution across varieties and sectors at their baseline calibration and markup elasticity at empirically supported levels. In our baseline model with Dixit-Stiglitz (1977) CES demand aggregation, no strategic complementarities and Calvo (1983) type price setting behavior, the markup over marginal cost is constant and hence the only source of persistence is firms' inability to change their prices whenever they want. Incorporating the variable markup channel of real rigidities into our baseline model is expected to add another source of aggregate RER persistence following a shock to nominal aggregate demand. Since the endogenous markup is decreasing in the relative price of a firm, the incentives of a firm to reset its price would be reduced further with variable markups, unless its opponents change their prices too.

In the literature, there are several ways to introduce variable markups into the standard sticky price models. Some of the recent studies are explored in the upcoming Literature Review section. Among them, three main approaches stand out: (i) using translog expenditure function introduced by Feenstra (2003), (ii) introducing strategic complementarities in price setting between large firms into a model with CES aggregation such as in Atkeson and Burstein (2008), and (iii) using a non-CES demand for intermediate goods such as the one introduced by Kimball (1995). In this paper we follow the third approach as it fits the best with our baseline model.

Even though we use a constant returns to scale (CRS) calibration similar to that in Kehoe and Midrigan (2007) as the baseline, our findings are quite consistent with that of Carvalho and Nechio (2011), whose baseline calibration has a DRS production technology. First of all, counterfactual one sector economy is unable to produce realistic levels of aggregate RER persistence even under extremely high degrees of aggregate nominal demand persistence. On the other hand, the multisector model with heterogeneity in frequency of price adjustment generates realistic aggregate RER dynamics when the nominal aggregate demand persistence is set to the level that is commonly used in literature. Compared to one sector counterfactual economy, the heterogeneity in frequency of price adjustment, coupled with pricing in local currency amplify the aggregate

RER persistence in response to a shock to aggregate nominal demand. This result is true regardless of the degree of nominal aggregate demand persistence.

When we use the median of the distribution of frequency of price adjustment rather than the weighted average, we see that one sector economy does significantly better in explaining the RER dynamics. Yet, the counterfactual one sector economy falls considerably short of heterogenous multisector economy in matching the RER persistence seen in data. Furthermore, we also created an artificial economy with only three sectors. This economy is calibrated by matching the first and second moments of frequency of price adjustment and average duration of spell. This model provides a reasonably good approximation of our sixty-seven sector model and thus, saves big time in computation.

Introducing variable markup channel of real rigidities into our baseline model increases the aggregate RER persistence further as expected and closes some of the gap between the model and data. However, unless markup elasticity is set at unrealistically high levels (which corresponds to unreasonably high levels of superelasticity and thus, very large curvature of demand function), the model with variable markups is also unable to generate reasonable RER persistence for a level of aggregate demand persistence matching the dynamics of nominal exchange rates. Yet, despite its inability to reconcile dynamics of nominal and real exchange rates, the model with variable markups is still better in explaining the RER dynamics than our baseline model with CES demand since it relies less on degree of aggregate persistence.

The amplification of aggregate RER persistence stemming from the variable markups are even smaller under DRS calibration of Carvalho and Nechio (2011). This stems from the fact that, under DRS, there are two countereffects when variable markups are introduced. In one hand, wage effect as named by Chari, Kehoe and McGrattan (2000) reduces the incentives to increase price, since marginal cost is now multiplied by a smaller multiplier. On the other hand, output effect increases the marginal cost and thus firms are more willing to increase their prices. Hence, when variable markups are introduced, the improvement in aggregate RER persistence is smaller under DRS case than CRS case. This will be discussed in more detail in the upcoming sections.

Based on these, we come up with the conclusion that empirically calibrated variable markup channel of real rigidities goes a modest way in amplifying the aggregate RER persistence and as exogenous persistence gets smaller, the absolute effect also gets less significant. In other words, despite the improvement, some degree of aggregate demand persistence is needed to generate realistic RER dynamics. Still, we can conclude that model with variable markups is better in explaining the RER dynamics compared to the baseline model, hence it should be preferred. However, we should also look for other sources of real rigidities. At this point, the first candidate would be introducing sector specific capital. However, in a recent ongoing paper, Carvalho and Nechio (2012) nest sector specific capital to their model. To our surprise, they find that even though incorporating capital into the baseline model increases RER persistence, making it sector specific reduces it. Similarly, in an extension to their model, Kehoe and Midrigan (2007) change their basic setup and allow the intermediate firms to use intermediate goods as an input to their production. They also find that this form of real rigidity does not increase the RER persistence.

Because the improvement we achieved through introducing variable markups is moderate, our findings seem inconsistent with the predictions of Carvalho and Nechio (2011) and confirm more the other studies such as Kehoe and Midrigan (2007), Chari, Kehoe and McGrattan (2000) and Gopinath and Itskhoki (2010a) among many others. There are mainly two reasons for the gap between our results and the predictions of Carvalho and Nechio (2011). First of all, they employ unrealistically high levels of elasticities of substitution across varieties and sectors. If we had also set markup elasticity to an empirically unreasonable level, we would obtain a much higher degree of RER persistence as well. However, we kept markup elasticity (and hence superelasticity) at an empirically documented level. More importantly, Carvalho and Nechio (2011) deviate from their baseline calibration when they try to predict the effects of real rigidities in their model and compare this to their baseline model. However, those two calibrations are not comparable since this new calibration already generates significantly larger persistence at every levels of elasticities of substitution and nominal aggregate demand persistence. We could have generated similar results by reverting to their alternative calibration. However, this would not change our conclusion that variable markups alone are not enough to reconcile the aggregate RER dynamics with nominal exchange rate dynamics, even though they moderately bring those two closer.

The paper is organized as follows: Section 2 will provide a brief literature review over the papers that we benefited the most. The literature on real exchange rates is enormous and thus we do not claim here to cover a considerable part of it. The benchmark model and its extensions along with the solution method and assumptions will be described in Section 3. Section 4 will present the parameterization of the models, while the quantitative findings will be discussed in Section 5. Finally, section 6 will conclude.

2 LITERATURE REVIEW

Our paper is related to more than one literature. First of all, there is a vast literature on determinants of exchange rates and PPP puzzle. Even though we are not exploring most of them here, a detailed review of studies on PPP can be found at Rogoff (1996) and Taylor and Taylor (2004). Not necessarily related to real exchange rates, there are also many studies nesting real rigidities into standard sticky price models. These models are used to explain various subjects such as gains from trade, international relative prices, monetary non-neutrality and dynamics of exchange rates among many others. Our paper have ingredients from both of these literatures. Here, we will briefly discuss some recent papers that are most related and were beneficial to ours. Hence, our literature review is not exhaustive.

The baseline model we used in this paper is mainly based on the model of Chari, Kehoe and McGrattan (2002) except for the lack of heterogeneity in their model. They come up with the conclusion that if prices are held fixed for at least one year (in our case average duration is only 4.7 months), risk aversion is high, and preferences are separable in leisure, then real exchange rates generated by the model are as volatile as in the data and quite persistent, but less so than in the data. They attribute the inability of the model to replicate the US data to the unrealistically high correlation between RERs and the ratio of consumption across countries. Our model share this property as well, which is inconsistent with data. However, we try to increase the persistence of the model first by introducing heterogeneity and later by incorporating the variable markups into the baseline model of Chari, Kehoe and McGrattan (2002).

In a similar model, but with variable desired markups a la Kimball (1995), Bouakez (2005) show that variable markups exacerbate the effects of nominal rigidity in terms of generating a reasonably high level of RER persistence. His model, which is estimated by the maximum-likelihood method using Canadian and U.S. data, quite successfully matches the persistence found in the real exchange rate series between Canada and US.

Kehoe and Midrigan (2007) have a similar approach to ours in the sense that they extend the commonly used sticky price models to have heterogeneity in the degree of price stickiness. They mainly focus on RER persistence at the good level across countries. They also conduct panel data analysis to compare the theoretical results with data. Their findings are not very consistent with the data especially when the magnitudes are concerned. In one sense, they find support for the view that the higher the price stickiness, the more persistent the RER of a good is. However, they also find that the discrepancy between the theory and data is very large quantitatively. Theory predicts that the RER persistence should range from fairly transient to fairly persistent, depending on the degree of price stickiness. Nonetheless, in data, almost all goods have persistent RERs, which is attributed to the fact that individual RERs tend closely track the nominal exchange rate, regardless of price stickiness. Just like we do in this paper, they also add price complementarities (through the use of intermediate goods as inputs in production) to their baseline model. However, they reach to the conclusion that adding real rigidities leads to only a very slight improvement. Carvalho and Nechio (2011) explain this by the very low level of exogenous aggregate demand persistence in Kehoe and Midrigan (2007).

Unlike our Calvo (1983) type price stickiness, Nakamura and Steinsson (2010) calibrate a multi sector menu cost model with the same data we used in this paper. They also augment the baseline menu cost model to incorporate intermediate inputs in production. The aim of their paper is to generate a level of monetary non-neutrality in response to nominal shocks, that is consistent with the US data. Similar to our paper, they also find that the introduction of heterogeneity in frequency of price adjustment almost triples the degree of monetary non-neutrality. Furthermore, they also show that the introduction of intermediate inputs raises the degree of monetary non-neutrality by another factor of three, without adversely affecting the model's ability to match the large average size of price changes. Another important finding is that a one sector economy that is calibrated to have frequency of price adjustment of the median (rather than mean) of the multi sector model would generate a similar level of monetary non-neutrality. Overall, their model shows that aggregate nominal shocks can account for around 23 percent of US business cycles, which is close to one-third observed in data.

This paper is most related with the work of Carvalho and Nechio (2011). Using the data of Nakamura and Steinsson (2008), they aggregate 271 sectors into 67 sectors and calculate their frequency of price adjustment and sectoral weights. The model used in their paper is that of Chari, Kehoe and McGrattan (2002) except for the heterogeneity introduced. Comparing the RER persistence and volatility of the multisector model with that of a counterfactual one sector economy, they find that deviations from PPP are more volatile and persistent than in an otherwise identical one-sector world economy with the same average frequency of price changes. However, the success of their model relies on a high degree of aggregate nominal demand persistence and it fails for lower level of demand persistence. The authors attribute the inability of their model to generate high enough persistence at a level of aggregate demand persistence consistent with nominal exchange rate movements to the lack of real rigidities in their model other than DRS production function. However, in a recent and unpublished paper, Carvalho and Nechio (2012) show that real rigidities introduced in the form of sector specific capital actually reduce persistence rather than increasing it. Finally, Carvalho and Nechio (2011) also decompose the persistence into two components: aggregation and heterogeneity effects. They reconcile the conflicting views of different papers saying the heterogeneity and aggregation plays a small role (Chen and Engel, 2005 and Crucini and Shintani, 2008) and the ones saying heterogeneity can explain most of the aggregate RER persistence (Imbs, Mumtaz, Ravn and Ray, 2005). Their conclusion is that these papers are not really conflicting but simply focusing on different sources of persistence and it is the heterogeneity effect that has the largest effect while the effect of aggregation is quite small.

In a closed economy set up, Eichenbaum and Fischer (2004) extends a standard version of Calvo (1983) type sticky price model in two ways. First, they introduce variable elasticity of demand of the differentiated goods by employing a Kimball (1995) type aggregator rather than CES aggregation. Secondly, they also introduce imperfect capital mobility, meaning that capital is fixed and cannot be reallocated instantly. Since their model is a closed economy one, they focus on the persistence of price rigidities, which would be related to RER rigidity in an open economy model. Similar to other studies in the literature, Eichenbaum and Fischer (2004) also find that standard Calvo models are not capable of generating a high level of price rigidity consistent with data. However, once variable markups and immobile capital are introduced, more reasonable results are obtained in terms of price rigidity. Their results are consistent with that of a later study by Woodford (2005) who also investigates the frequency of price adjustment implied by a Calvo model with variable markups and firm-specific capital.

In an another closed economy environment, Klenow and Willis (2006) evaluate the variable markup channel of real rigidities by calibrating a menu cost model with Kimball (1995) aggregator to match the micro data from US CPI in terms of the frequency and size of price adjustment. They reach to the conclusion that the real rigidities alone are not sufficient to generate realistic levels of persistence. Once a significantly large degree of real rigidities are imposed to generate considerable monetary non-neutrality, implausibly large size of idiosyncratic shocks, large movements in micro quantities and very high levels of menu costs are required. Gopinath and Itskhoki (2010a) provide the intuition for this outcome: Significant real rigidities compress price dispersion so that much larger idiosyncratic shocks are required to match the size of price adjustment. At the same time, large menu costs are required to match the average durations of price rigidity, given large idiosyncratic shocks.

Rather than using a Kimball (1995) aggregator, Burstein and Hellwig (2007) introduce real rigidities by increasing marginal costs at the firms level, which generates strategic complementarity in price setting. They calibrate a closed economy, menu cost model of price adjustment matching the degree of decreasing returns to scale using scanner data on prices and market shares at the level of individual varieties. They find that strategic complementarities play a very moderate role in generating large aggregate real effects from the nominal shocks.

Both the closed and open economy literatures on real rigidities are brought together in Gopinath and Itskhoki (2010a). They evaluate both Calvo and menu cost models with variable markups at the wholesale level and constant markups at the retail level and try to match the data on international prices and exchange rate shocks. They show that variable markups alone generate price sluggishness at the aggregate level, while they fall short of matching price persistence at the micro level. As in most of the studies in the literature, the absolute effects of variable markups are modest unless they are coupled with exogenous sources of persistence. In another paper, Gopinath and Itskhoki (2010b) evaluate a dynamic menu cost model with Kimball (1995) demand and they find that the variable markup channel can generate significant variation in frequency of price adjustment, which can be translated into heterogeneity in price adjustment and hence larger aggregate persistence.

Alternative to Kimball (1995) type of demand, Atkeson and Burstein (2008) develop a model with Cournot competition and CES demand function to generate variable markups. In this model, strategic interactions among the large firms affect the level of markups through the market shares. They mainly focus on the dynamics of international pricing rather than real exchange rates. They find that, when their model is parameterized to match the data on international trade and market structure in the US, the deviations from the relative PPP can be reproduced. They also assess the extent to which imperfect competition with variable markups and international trade costs play essential roles in generating these results and come up with the conclusion that these features are key in producing their results.

There are several alternative methods of incorporating variable markups into otherwise standard models. Among them, some recent ones worth mentioning here even though they are not directly related to the PPP literature. Recently, Arkolakis, Costinot, Donaldson and Rodriguez-Clare (2012) extend Arkolakis, Costinot and Rodriguez-Clare (2011) and investigate the gains from trade by introducing variable markups to the original model with constant markups. They consider three main alternatives to their baseline CES demand function: separable but non-CES utility functions; a quadratic, but non-separable utility function; and a translog expenditure function as in Feenstra (2003), Bergin and Feenstra (2009) and Feenstra and Weinstein (2010). Skipping the details, they find that gains from trade liberalization are weakly lower than those predicted by models with constant markups. They also show that distribution of markups are invariant to openness to trade, even though markups vary across firms. In another study that employs translog expenditures, Bilbiie, Ghironi and Melitz (2012) model endogenous entry/exit and hence product varieties to analyze the macroeconomic fluctuations. They find that the sluggish response of the number of producers generates a new and potentially important endogenous propagation mechanism for business cycle models. As a matter of fact, their model is able to replicate several features of business cycles. Finally, Gopinath and Itskhoki (2010a) introduce an alternative setup in which final good producers bargain with a number of intermediate good suppliers. Their model can be considered as a general case of Atkeson and Burstein (2008). They show that in a static bargaining model there are strategic complementarities in wholesale price setting, so that markups are variable and wholesale prices exhibit incomplete pass-through, while final good producers still have constant markups.

In summary, most of the studies show that the models with variable markups are more successful than the models with constant markups in replicating the persistence in RER, international relative prices and other aspects of macroeconomic fluctuations. Nevertheless, in most of these studies, the contribution of the variable markups still remains relatively modest. Hence, introducing variable markups alone does not seem sufficient as a source of real rigidities. This invokes the incorporation of other sources of real rigidities along with variable markups. Furthermore, there does not exist many studies introducing heterogeneity in variability of markups (other than broad heterogeneity of Gopinath and Itskhoki (2010a), who only differentiate between wholesale and retail sectors) and hence this seems like a research agenda that is worth being pursued in the future.

3 MODELS

In this section, we will first introduce our baseline model with Dixit-Stiglitz aggregation. The model mainly follows Chari, Kehoe and McGrattan (2002) and Carvalho and Nechio (2011). The notation is also borrowed from these two papers. Later, we will introduce a model with variable markups using a non-CES aggregation at the intermediate good production level. To that end, we will introduce a variant of Kimball (1995) aggregator as in Klenow and Willis (2006) into our baseline model. The ingredients of this extended model are mainly from Eichenbaum and Fisher (2004) and Smets and Wouters (2007) as well as Gopinath and Itskhoki (2010a and 2010b).

3.1 Baseline Model: Constant Markups

The world economy consists of two symmetric countries: *Home* and *Foreign*.[4] In both countries, there exists a large number of identical and infinitely lived consumers, who supply their labor to intermediate good producing firms that they own. Using this labor as the only factor of production, the monopolistically competitive firms produce a variety of the intermediate good, which is the only traded item in the model. These firms can price discriminate across countries and set their prices in local currency in a staggered fashion. Heterogeneity is introduced by dividing the varieties into sectors that differ in their frequency of price adjustment. Once the price is set, the forthcoming demand should be satisfied at that price level. There are also competitive final good producers in each country that purchase intermediate goods from both countries and aggregate the varieties into a non-tradable final good, which is used only for consumption. As it is convenient in the literature, we will follow Woodford (2003) and work with the cashless limit of a monetary economy.

[4]Throughout the paper, foreign variables are denoted with superscript $*$.

3.1.1 Households

Households supply labor to the intermediate good producing firms that they own and they consume the non-tradable final good. They also engage in trading state contingent bonds internationally and receive dividend payments from the monopolistically competitive intermediate good producers. Given these, the representative household in the home country solves the following problem:

$$\max_{C_t,L_t,B_{t+1}} E_0\sum_{t=0}^{\infty}\beta^t(\frac{C_t^{1-\sigma}-1}{1-\sigma}-\frac{L^{1+\gamma}}{1+\gamma})$$

subject to the following flow budget constraint:

$$P_tC_t+E_t(Q_{t,t+1}B_{t,t+1})\leq W_tL_t+B_t+\Pi_t \tag{1}$$

and the transversality condition:

$$\lim_{s\to\infty}E_t(Q_{t,t+s}B_{t+s})=0 \tag{2}$$

where, C_t is consumption, L_t is labor supply, P_t is price of final good in home country, W_t is the nominal wage rate at the market and Π_t is the dividend payments received from the intermediate good producing firms at time t. B_{t+1} , on the other hand, represents the level of state contingent claims on a portfolio of financial securities that the household chooses at period t to enter the next period. Due to the complete markets assumption, the households can choose a different value of B_{t+1} for each possible state of the world at all times. $Q_{t,t+1}$ is the stochastic discount factor implied by a no-arbitrage condition and it can be considered as the current price of the bond holdings next period. Finally, considering the parameters, β is the usual discount factor, $\frac{1}{\sigma}$ is the intertemporal elasticity of substitution and $\frac{1}{\gamma}$ is the Frisch elasticity of substitution.

The First Order Conditions (FOCs) of this problem are as follows:

$$\begin{aligned}
C_t &: \quad C_t^{-\sigma}=\lambda_tP_t \\
L_t &: \quad L_t^{\gamma}=\lambda_tW_t \\
B_{t+1} &: \quad \lambda_tQ_{t,t+s}=\lambda_{t+s}\beta^s
\end{aligned}$$

where λ_t is the Lagrangian multiplier for the flow budget constraint. Plugging the FOC for C_t into the other two, we obtain:

$$
\begin{aligned}
L_t^\gamma C_t^\sigma &= \frac{W_t}{P_t} && (3)\\
Q_{t,t+s} &= \beta^s(\frac{C_t}{Ct+s})^\sigma \frac{P_t}{P_{t+s}} && (4)
\end{aligned}
$$

Here, $Q_{t,t} = 1$ and $Q_{t,t+s} = \prod_{r=t+1}^{t+s} Q_{r-1,r}$ for $s > 0$.

The problem for the foreign consumer is the same. However, we amend the budget constraint as follows:

$$
P_t^* C_t^* + E_t(Q_{t,t+1}^* \frac{B_{t,t+1}^*}{e_t}) \leq W_t^* L_t^* + \frac{B_t^*}{e_t} + \Pi_t^* \qquad (5)
$$

Here, e_t represents the bilateral nominal exchange rate defined as the home currency price of one unit of foreign currency. Underlying this budget constraint, there is the implicit assumption that all state contingent bonds are denominated in home currency and hence it needs to be converted to foreign currency when constructing the foreign budget constraint.

FOCs for the foreign consumers' problem gives:

$$
\begin{aligned}
L_t^{*\gamma} C_t^{*\sigma} &= \frac{W_t^*}{P_t^*} && (6)\\
Q_{t,t+s}^* &= \beta^s(\frac{C_t^*}{C^*t+s})^\sigma \frac{P_t^*}{P_{t+s}^*}\frac{e_t}{e_{t+s}} && (7)
\end{aligned}
$$

Because the state contingent bonds are traded internationally and due to the no arbitrage condition, the stochastic discount factor should be the same in both countries. As a consequence, the right hand sides of equations (4) and (7) should equal to each other at every possible state:

$$
(\frac{C_t}{Ct+s})^\sigma \frac{P_t}{P_{t+s}} = (\frac{C_t^*}{C^*t+s})^\sigma \frac{P_t^*}{P_{t+s}^*}\frac{e_t}{e_{t+s}} \qquad (8)
$$

Rearranging equation (8) and defining $q_t = e_t \frac{P_t^*}{P_t}$ as the real exchange rate (RER) between two countries, we obtain:

$$q_{t+s} = q_t(\frac{C_t^*}{C^*t+s})^{\sigma}(\frac{C_t}{Ct+s})^{-\sigma} \tag{9}$$

Following Carvalho and Nechio (2011), we set $q_0 = 1$ and iterate equation (9) backward, we obtain the relationship between RER and consumption levels in both countries:

$$q_t = (\frac{C_t}{C_t^*})^{\sigma} \tag{10}$$

Equation (10) implies that RER and consumption ratio in two countries are perfectly correlated. Even though this is not consistent with data, it is a common characteristics of most of the models in literature that have full risk sharing through state contingent bonds.

3.1.2 Firms

There is one single final non-tradable consumption good, that is produced in both countries. This good is produced by competitive firms that aggregate K different intermediate goods from both countries. The sectoral division of the monopolistically competitive intermediate good producers are based on their frequency of price adjustment. There is a continuum of firms in every sector and hence none of them have a significant market share. The firms are indexed according to the country of origin: H and F, their sector: $k \in \{1, 2, ..., K\}$ and further by the variety they produce within their sector: $j \in [0, 1]$. The firms are distributed across sectors according to the weights f_k such that $\sum_{k=1}^{K} f_k = 1$ and $f_k > 0$.

We have three levels of aggregation for each country in the model. First of all, varieties in each sector that are produced at home ($Y_{H,k,j,t}$) and foreign country ($Y_{F,k,j,t}$) and sold at home are aggregated to obtain the sectoral levels of home ($Y_{H,k,t}$) and foreign ($Y_{F,k,t}$) goods sold at home respectively (equations 11 and 12). Secondly, home and foreign intermediate goods of each sector sold at home are aggregated to give the total output of each sector sold at home ($Y_{k,t}$, equation 13). Finally, home final good (Y_t) is the aggregation of output of each sector sold at home (equation 14).

$$Y_{H,k,t} = (f_k^{(\theta-1)/\theta} \int_0^1 Y_{H,k,j,t}^{(\theta-1)/\theta} dj)^{\theta/(\theta-1)} \quad (11)$$

$$Y_{F,k,t} = (f_k^{(\theta-1)/\theta} \int_0^1 Y_{F,k,j,t}^{(\theta-1)/\theta} dj)^{\theta/(\theta-1)} \quad (12)$$

$$Y_{k,t} = (\omega^{1/\rho} Y_{H,k,t}^{(\rho-1)/\rho} + (1-\omega)^{1/\rho} Y_{F,k,t}^{(\rho-1)/\rho})^{\rho/(\rho-1)} \quad (13)$$

$$Y_t = (\sum_{k=1}^{K} f_k^{1/\eta} Y_{k,t}^{(\eta-1)/\eta})^{\eta/(\eta-1)} \quad (14)$$

Here, $1-\omega \in [0,1]$ represents the steady state share of imports used in production. $\theta > 1$ is the elasticity of substitution between varieties within sectors, $\rho \geq 0$ is the elasticity of substitution between home and foreign goods and finally $\eta \geq 0$ is the elasticity of substitution across sectors. The foreign counterparts of these equations are as follows:

$$Y_{H,k,t}^{*} = (f_k^{(\theta-1)/\theta} \int_0^1 Y_{H,k,j,t}^{*(\theta-1)/\theta} dj)^{\theta/(\theta-1)} \quad (15)$$

$$Y_{F,k,t}^{*} = (f_k^{(\theta-1)/\theta} \int_0^1 Y_{F,k,j,t}^{*(\theta-1)/\theta} dj)^{\theta/(\theta-1)} \quad (16)$$

$$Y_{k,t}^{*} = ((1-\omega)^{1/\rho} Y_{H,k,t}^{*(\rho-1)/\rho} + \omega^{1/\rho} Y_{F,k,t}^{*(\rho-1)/\rho})^{\rho/(\rho-1)} \quad (17)$$

$$Y_t^{*} = (\sum_{k=1}^{K} f_k^{1/\eta} Y_{k,t}^{*(\eta-1)/\eta})^{\eta/(\eta-1)} \quad (18)$$

Final Good Producers The problem of the representative home country final good producer is:

$$\max_{Y_{h,k,j,t}, Y_{F,k,jt}} P_t Y_t - \sum_{k=1}^{K} f_k \int_0^1 (P_{H,k,j,t} Y_{H,k,j,t} + P_{F,k,j,t} Y_{F,k,j,t}) dj$$

subject to (11), (12), (13) and (14).

Here, P_t is the domestic price of final good, $P_{H,k,j,t}$ is the price charged at home at time t by firm j in sector k by a home firm. $P_{F,k,j,t}$ is the same for a foreign firm that sells in home country and hence in home currency.

This problem has a standard, three step Dixit-Stiglitz solution. The resulting demand functions for variety j in sector k,sold in home country and produced by home (equation 19) and foreign (equation 20) firms are:

$$Y_{H,k,j,t} = \omega(\frac{P_{H,k,j,t}}{P_{H,k,t}})^{-\theta}(\frac{P_{H,k,t}}{P_{k,t}})^{-\rho}(\frac{P_{k,t}}{P_t})^{-\eta}Y_t \quad (19)$$

$$Y_{F,k,j,t} = (1-\omega)(\frac{P_{F,k,j,t}}{P_{F,k,t}})^{-\theta}(\frac{P_{F,k,t}}{P_{k,t}})^{-\rho}(\frac{P_{k,t}}{P_t})^{-\eta}Y_t \quad (20)$$

where, $P_{H,k,t}$ is the index for price charged at period t in sector k for goods produced and sold at home, $P_{F,k,t}$ is the same for the goods sold in home but produced in foreign country. Finally, $P_{k,t}$ is the index for price charged for sector k goods at home.

As it is standard in the literature, this process generates the following price equations:

$$P_{H,k,t} = (\int_0^1 P_{H,k,j,t}^{1-\theta}dj)^{1/(1-\theta)} \quad (21)$$

$$P_{F,k,t} = (\int_0^1 P_{F,k,j,t}^{1-\theta}dj)^{1/(1-\theta)} \quad (22)$$

$$P_{k,t} = (\omega P_{H,k,t}^{1-\rho} + (1-\omega)P_{F,k,t}^{1-\rho})^{1/(1-\rho)} \quad (23)$$

$$P_t = (\sum_{k=1}^{K} f_k P_{k,t}^{1-\eta})^{1/(1-\eta)} \quad (24)$$

The final good producers in foreign country solve a similar problem subject to equations (15) – (18). The resulting demand functions are as follows:

$$Y^*_{F,k,j,t} = \omega(\frac{P^*_{F,k,j,t}}{P^*_{F,k,t}})^{-\theta}(\frac{P^*_{F,k,t}}{P^*_{k,t}})^{-\rho}(\frac{P^*_{k,t}}{P^*_t})^{-\eta}Y^*_t \quad (25)$$

$$Y^*_{H,k,j,t} = (1-\omega)(\frac{P^*_{H,k,j,t}}{P^*_{H,k,t}})^{-\theta}(\frac{P^*_{H,k,t}}{P^*_{k,t}})^{-\rho}(\frac{P^*_{k,t}}{P^*_t})^{-\eta}Y^*_t \quad (26)$$

Defining P_t^*, $P_{k,t}^*$, $P_{H,k,t}^*$, $P_{F,k,t}^*$, $P_{H,k,j,t}^*$ and $P_{F,k,j,t}^*$ same as their counterparts we have the following standard price indices for the foreign country:

$$P_{H,k,t}^* = (\int_0^1 P_{H,k,j,t}^{*(1-\theta)} dj)^{1/(1-\theta)} \tag{27}$$

$$P_{F,k,t}^* = (\int_0^1 P_{F,k,j,t}^{*(1-\theta)} dj)^{1/(1-\theta)} \tag{28}$$

$$P_{k,t}^* = ((1-\omega)P_{H,k,t}^{*(1-\rho)} + \omega P_{F,k,t}^{*(1-\rho)})^{1/(1-\rho)} \tag{29}$$

$$P_t^* = (\sum_{k=1}^{K} f_k P_{k,t}^{*(1-\eta)})^{1/(1-\eta)} \tag{30}$$

Price Setting by Intermediate Good Producers Nominal rigidities are introduced into the model through Calvo (1983) type price setting behavior for the sake of tractability. Every period, each firm j in sector k changes its price with a constant probability, α_k. Thus, on average the firms in sector k change their prices once in every $1/\alpha_k$ periods. As common in the literature, firms' ability to reset their prices is independent across firms within a sector and time.

At this point, we make the assumption that if an intermediate firm has the opportunity to reset its price, it will do so in both markets. Hence, a home intermediate firm j in sector k will choose two prices at the same time: $X_{H,k,j,t}$ and $X_{H,k,j,t}^*$, which are the price charged in home country in home currency and price charged in foreign country in foreign currency respectively. In this framework, a home country intermediate good firm solves the following maximization problem:

$$\max_{X_{H,k,j,t}, X_{H,k,j,t}^*, N_{k,j,t}} E_t Q_{t,t+s}(1-\alpha_k)^s \begin{bmatrix} X_{H,k,j,t} Y_{H,k,j,t+s} \\ +e_{t+s} X_{F,k,j,t}^* Y_{h,k,j,t+s}^* - W_{t+s} L_{k,j,t+s} \end{bmatrix}$$

subject to (19) and (26) and

$$Y_{H,k,j,t} + Y_{H,k,j,t}^* = L_{k,j,t}^{\chi} \tag{31}$$

First order conditions of this problem are:[5]

$$\begin{aligned} X_{H,k,j,t} &: \quad E_t Q_{t,t+s}(1-\alpha_k)^s(((1-\theta)X_{H,k,j,t}^{-\theta} - \theta\frac{W_{t+s}}{\chi L_{k,j,t+s}^{\chi-1}}X_{H,k,j,t}^{-\theta-1})\Lambda_{H,k,t+s} = 0 \\ X_{H,k,j,t}^{*} &: \quad E_t Q_{t,t+s}(1-\alpha_k)^s(((1-\theta)e_{t+s}X_{H,k,j,t}^{*-\theta} - \theta\frac{W_{t+s}}{\chi L_{k,j,t+s}^{\chi-1}}X_{H,k,j,t}^{*-\theta-1})\Lambda_{H,k,t+s}^{*} = 0 \end{aligned}$$

where,

$$\begin{aligned} \Lambda_{H,k,t+s} &= \omega(\frac{1}{P_{H,k,t+s}})^{-\theta}(\frac{P_{H,k,t+s}}{P_{k,t+s}})^{-\rho}(\frac{P_{k,t+s}}{P_{t+s}})^{-\eta} = 0 \\ \Lambda_{H,k,t+s}^{*} &= (1-\omega)(\frac{1}{P_{H,k,t+s}^{*}})^{-\theta}(\frac{P_{H,k,t+s}^{*}}{P_{k,t+s}^{*}})^{-\rho}(\frac{P_{k,t+s}^{*}}{P_{t+s}^{*}})^{-\eta} = 0 \end{aligned}$$

Rearranged, these first order conditions result in the following price setting equations:

$$X_{H,k,j,t} = \frac{\theta}{\theta-1}\frac{\sum_{s=0}^{\infty} Q_{t,t+s}(1-\alpha_k)^s\Lambda_{H,k,t+s}\frac{W_{t+s}}{\chi L_{k,j,t+s}^{\chi-1}}}{\sum_{s=0}^{\infty} Q_{t,t+s}(1-\alpha_k)^s\Lambda_{H,k,t+s}} \tag{32}$$

$$X_{H,k,j,t}^{*} = \frac{\theta}{\theta-1}\frac{\sum_{s=0}^{\infty} Q_{t,t+s}(1-\alpha_k)^s\Lambda_{H,k,t+s}^{*}\frac{W_{t+s}}{\chi L_{k,j,t+s}^{\chi-1}}}{\sum_{s=0}^{\infty} Q_{t,t+s}(1-\alpha_k)^s e_{t+s}\Lambda_{H,k,t+s}} \tag{33}$$

Here, $\frac{W_{t+s}}{\chi L_{k,j,t+s}^{\chi-1}}$ is the marginal cost of production. Equations (32) and (33) are the standard Calvo price setting equations showing that once a firm has the opportunity to change its price, it takes all the future values of marginal cost into account. $\mu = \frac{\theta}{\theta-1} > 1$ is the *constant markup* over the marginal cost and hence, as we show in Appendix A , it disappears after linearization. This markup is the same for every firm in every sector. This proves that the CES aggregation we use in the baseline model generates a constant markup.

An analogous maximization problem yields similar equations for the foreign firms:

[5] Here we plugged $\gamma_{t+s} = \frac{W_{t+s}}{N_{k,j,t+s}}$, where γ_{t+s} is the Lagrangean multiplier for the constraint (31).

$$X^*_{F,k,j,t} = \frac{\theta}{\theta-1}\frac{\sum_{s=0}^{\infty} Q_{t,t+s}(1-\alpha_k)^s \Lambda^*_{F,k,t+s}\frac{W^*_{t+s}}{\chi L^{*\chi-1}_{k,j,t+s}}}{\sum_{s=0}^{\infty} Q_{t,t+s}(1-\alpha_k)^s \Lambda^*_{F,k,t+s}} \quad (34)$$

$$X_{F,k,j,t} = \frac{\theta}{\theta-1}\frac{\sum_{s=0}^{\infty} Q_{t,t+s}(1-\alpha_k)^s \Lambda^*_{F,k,t+s}\frac{W^*_{t+s}}{\chi L^{*\chi-1}_{k,j,t+s}}}{\sum_{s=0}^{\infty} Q_{t,t+s}(1-\alpha_k)^s \frac{\Lambda^*_{H,k,t+s}}{e_{t+s}}} \quad (35)$$

where,

$$\Lambda_{F,k,t+s} = (1-\omega)(\frac{1}{P_{F,k,t+s}})^{-\theta}(\frac{P_{F,k,t+s}}{P_{k,t+s}})^{-\rho}(\frac{P_{k,t+s}}{P_{t+s}})^{-\eta} = 0$$

$$\Lambda^*_{H,k,t+s} = \omega(\frac{1}{P^*_{F,k,t+s}})^{-\theta}(\frac{P^*_{F,k,t+s}}{P^*_{k,t+s}})^{-\rho}(\frac{P^*_{k,t+s}}{P^*_{t+s}})^{-\eta} = 0$$

and $\frac{W^*_t}{\chi L^{*\chi-1}_{k,j,t}}$ is the foreign marginal cost.

In a steady state equilibrium, all firms in a sector that set a new price would set the same price since they will face the same information set and the joint distribution of future variables that matter for price setting would be the same for all firms in sector k (Carvalho and Nechio, 2011). Therefore, we drop the j subscript and denote the reset prices as $X_{H,k,t}$, $X^*_{H,k,t}$, $X_{F,k,t}$ and $X^*_{F,k,t}$. Then, the sectoral country price indices can be re-written as:

$$P_{H,k,t} = (\alpha_k X^{1-\theta}_{H,k,t} + (1-\alpha_k)P^{1-\theta}_{H,k,t-1})^{1/(1-\theta)} \quad (36)$$

$$P^*_{H,k,t} = (\alpha_k X^{*(1-\theta)}_{H,k,t} + (1-\alpha_k)P^{*(1-\theta)}_{H,k,t-1})^{1/(1-\theta)} \quad (37)$$

$$P^*_{F,k,t} = (\alpha_k X^{*(1-\theta)}_{F,k,t} + (1-\alpha_k)P^{*(1-\theta)}_{F,k,t-1})^{1/(1-\theta)} \quad (38)$$

$$P_{F,k,t} = (\alpha_k X^{1-\theta}_{F,k,t} + (1-\alpha_k)P^{1-\theta}_{F,k,t-1})^{1/(1-\theta)} \quad (39)$$

The assumption underlying the equations $(36)-(39)$ is that if a firm cannot reset its price, it keeps its price at its previous level rather than some sort of static or dynamic indexing, such as in Eichenbaum and Fisher (2004) and Smets and Wouters (2007).

3.1.3 Government

The baseline model is closed with a government that implements a monetary policy, which ensures existence and uniqueness of a rational-expectations equilibrium. As it is standard in the monetary economics literature, the specification of the monetary policy is left implicit and it is assumed that the growth rate of nominal aggregate demand in each country follows an exogenous first order autoregressive process. As Carvalho and Nechio (2011) state, this specification fits the economic data very well.

Let Z_t and Z_t^* denote the nominal aggregate demand in home and foreign countries respectively:

$$Z_t \equiv P_t Y_t \tag{40}$$
$$Z_t^* \equiv P_t^* Y_t^* \tag{41}$$

The assumption we made about the growth rate of nominal aggregate demand can be written formally as:

$$\Delta z_t = \rho_z \Delta z_{t-1} + \sigma_\varepsilon \varepsilon_{z,t} \tag{42}$$
$$\Delta z_t^* = \rho_z \Delta z_{t-1}^* + \sigma_\varepsilon \varepsilon_{z,t}^* \tag{43}$$

where, Δz_t and Δz_t^* are the growth rates of nominal aggregate demand in home and foreign countries respectively. The autocorrelation in nominal aggregate demand growth, ρ_z, is employed as the measure of aggregate persistence. Finally, $\varepsilon_{z,t}$ and $\varepsilon_{z,t}^*$ are uncorrelated, zero mean, unit variance and independently and identically distributed shocks.

3.2 Equilibrium[6]

An equilibrium for this economy is a collection of allocations for home consumers C_t, L_t and B_{t+1}; allocations for foreign consumers C_t^*, L_t^* and B_{t+1}^*; allocations and prices for home intermediate good producers $Y_{H,k,j,t}$, $Y_{H,k,j,t}^*$, $L_{k,j,t}$, $X_{H,k,j,t}$ and $X_{H,k,j,t}^*$ in each sector k; allocations and prices for foreign intermediate good producers $Y_{F,k,j,t}$, $Y_{F,k,j,t}^*$, $L_{k,j,t}^*$, $X_{F,k,j,t}^*$ and $X_{F,k,j,t}$in each sector k; sectoral allocations and prices for home and foreign goods $Y_{H,k,t}$, $Y_{F,k,t}$, $P_{H,k,t}$ and

[6]Equilibirum definition is based on Chari, Kehoe and McGrattan (2002) and Kehoe and Midrigan (2007).

$P_{F,k,t}$ sold in home country; sectoral allocations and prices for home and foreign goods $Y^*_{H,k,t}$, $Y^*_{F,k,t}$, $P^*_{H,k,t}$ and $P^*_{F,k,t}$ sold in foreign country; sectoral allocation of prices of intermediate goods in sold home country $Y_{k,t}$ and $P_{k,t}$; sectoral allocation of prices of intermediate goods sold in foreign country $Y^*_{k,t}$ and $P^*_{k,t}$; and allocations for home and foreign final goods producers Y_t, Y^*_t, P_t and P^*_t, real wages W_t, W^*_t, and bond prices $Q_{t,t+1}$ satisfying: (i) consumer allocations satisfy the consumers' problem in both countries; (ii) the prices of intermediate goods producers solve their maximization problem in both countries; (iii) the final goods producers' allocations solve their problem in both countries; (iv) the market clearing conditions hold; and (v) nominal aggregate demand satisfy the above specification.

3.3 Extended Model: Variable Markups

As shown in equations (32) – (35), the markup over marginal cost is constant and same across all firms within a sector, as well as across different sectors. This is the standard outcome of the CES aggregation in the absence of strategic complementarities. Introducing variable markups into our baseline model is expected to increase the RER persistence following a shock to nominal aggregate demand since the endogenous markup will be lower when the relative price of a firm is higher, hence reducing the incentive of the firm to reset its price further unless its opponents change their prices too.

There are several ways to introduce variable markups to our baseline model. Among them, three main approaches stand out: (i) using translog expenditure function introduced by Feenstra (2003)[7], (ii) introducing strategic complementarities in price setting between large firms into a model with CES aggregation such as Atkeson and Burstein (2008), and (iii) using a non-CES demand for intermediate goods such as the one introduced by Kimball (1995). Here, in this paper we will follow the third approach as it fits the best with our baseline model. Our non-CES aggregation model is mainly based on Woodford (2003), Klenow and Willis (2006), Eichenbaum and Fisher (2004), Smets and Wouters (2007) and Gopinath and Itskhoki (2010a).

We will introduce a non-CES aggregation of Kimball (1995) type only at the aggregation of intermediate good varieties at every sector. In other words, the aggregation of home and foreign goods sold at home/foreign country into total output of sector k sold at home/foreign country (equations 13 and 17), and aggregation of sectors into final good (equations 14 and 18) remain the same.

At time t, an intermediate good produced and sold at home in sector k is obtained by aggregating the varieties $j \in [0, 1]$ using the following technology suggested by Kimball (1995):

[7] Some recent studies using translog expenditure function of Feenstra (2003) are Arkolakis, Costinot and Donaldson (2012) and Bilbiie, Ghironi and Melitz (2012).

$$\int_0^1 G(\frac{Y_{H,k,j,t}}{Y_{H,k,t}})dj = 1 \quad (44)$$

Technology for $Y^*_{H,k,j,t}$, $Y_{F,k,j,t}$ and $Y^*_{F,k,j,t}$ are analogous. G is assumed to be increasing and strictly concave: $G'(.) > 0$, $G''(.) < 0$ and also $G(1) = 1$. The Dixit-Stiglitz (CES) in our baseline model corresponds to a special case of this:

$$G(\frac{Y_{H,k,j,t}}{Y_{H,k,t}}) = (\frac{Y_{H,k,j,t}}{Y_{H,k,t}})^{\frac{\theta-1}{\theta}} \quad (45)$$

and hence, our generalized Kimball (1995) aggregation encompasses the standard CES case as well.

The problem faced by the final good producers is the same as before, except for the constraints (11) and (12), which are now replaced by equation (44) and $\int_0^1 G(\frac{Y_{F,k,j,t}}{Y_{F,k,t}})dj = 1$. The resulting demand functions are:

$$Y_{H,k,j,t} = \omega\varphi(\frac{P_{H,k,j,t}}{P_{H,k,t}}D_{H,k,t})(\frac{P_{H,k,t}}{P_{k,t}})^{-\rho}(\frac{P_{k,t}}{P_t})^{-\eta}Y_t \quad (46)$$

$$Y_{F,k,j,t} = (1-\omega)\varphi(\frac{P_{F,k,j,t}}{P_{F,k,t}}D_{F,k,t})(\frac{P_{F,k,t}}{P_{k,t}})^{-\rho}(\frac{P_{k,t}}{P_t})^{-\eta}Y_t \quad (47)$$

Analogously, the foreign final good producers' problem gives the following demand functions:

$$Y^*_{F,k,j,t} = \omega\varphi(\frac{P^*_{F,k,j,t}}{P^*_{F,k,t}}D^*_{F,k,t})(\frac{P^*_{F,k,t}}{P^*_{k,t}})^{-\rho}(\frac{P^*_{k,t}}{P^*_t})^{-\eta}Y^*_t \quad (48)$$

$$Y^*_{H,k,j,t} = (1-\omega)\varphi(\frac{P^*_{H,k,j,t}}{P^*_{H,k,t}}D^*_{H,k,t})(\frac{P^*_{H,k,t}}{P^*_{k,t}})^{-\rho}(\frac{P^*_{k,t}}{P^*_t})^{-\eta}Y^*_t \quad (49)$$

where $\varphi(.) = G'^{-1}(.)$ is the inverse of the derivative of Kimball aggregator and $D_{H,k,t} = \int_0^1 G'(\frac{Y_{H,k,j,t}}{Y_{H,k,t}})\frac{Y_{H,k,j,t}}{Y_{H,k,t}}dj$. $D_{F,k,t}$, $D^*_{F,k,t}$ and $D^*_{H,k,t}$ are defined analogously.

Home and foreign sectoral price indices satisfy:

$$P_{H,k,t}Y_{H,k,t} = \int_0^1 P_{H,k,j,t}Y_{H,k,j,t}dj \tag{50}$$

since the aggregator (44) is homothetic. As usual, similar relations hold for other three related price indices.

The problem facing the intermediate good producers and our Calvo pricing set up is the same as before. However, we now use equations (46) and (49) as the constraints of the home producers and (47) and (48) of the foreign producers. The first order condition of the problem of a home firm with respect to $X_{H,k,j,t}$ is depicted below:

$$\sum_{s=0}^{\infty} Q_{t,t+s}(1-\alpha_k)^s Y_{H,k,j,t+s}\frac{D_{H,k,t+s}}{P_{H,k,t+s}}[X_{H,k,j,t}+(X_{H,k,j,t}-\frac{W_{t+s}}{\chi L_{k,j,t+s}^{\chi-1}})\frac{1}{\varphi(m_{t+s})}\frac{G'(x_{t+s})}{G''(x_{t+s})}] = 0 \tag{51}$$

where, $x_t = \varphi(m_t)$ and $m_t = \frac{P_{H,k,j,t}}{P_{H,k,t}}D_{H,k,t}$.

Manipulating this first order condition, marginal cost is multiplied by the term $A = \frac{G'(x_{t+s})}{\varphi(m_{t+s})G''(x_{t+s})+G'(x_{t+s})}$, rather than the constant multiplier $\frac{\theta}{\theta-1}$ under CES case. Thus, the markup over marginal cost is not a constant anymore and it is a decreasing function of the own relative price of a good. Therefore, we will have an additional term representing variable markups and this will not disappear after linearization. This coefficient can also be written as $A = \frac{1}{\frac{\varphi(m_{t+s})G''(x_{t+s})}{G'(x_{t+s})}+1}$.

We adopt the Kimball aggregator specification of Klenow and Willis (2006).[8] This results in:

$$G'^{-1}(x_j) = \varphi(x_j) = [1-\epsilon\ln(\frac{\theta x_j}{\theta-1})]^{\theta/\epsilon} \tag{52}$$

where $x_j \equiv D_{H,k,t}\frac{P_{H,k,j,t}}{P_{H,k,t}}$.

This demand representation is governed by two parameters: $\theta > 1$ -the elasticity of substitution across varieties within a sector as before-, and ϵ, which is the price elasticity of elasticity of demand and termed as *superelasticity*. Then, elasticity and super-elasticity are given as follows:

[8] However, it should be noted that we do not need to exactly specify the aggregator, since we will solve the model upto a first order approximation.

$$\widetilde{\theta}(x_j) = \frac{\theta}{1-\epsilon\ln(\frac{\theta x_j}{\theta-1})} \tag{53}$$

$$\widetilde{\epsilon}(x_j) = \frac{\epsilon}{1-\epsilon\ln(\frac{\theta x_j}{\theta-1})} \tag{54}$$

Note that when $\epsilon = 0$, we are back at CES case. Also, under CES case, $\varphi(x) = 1$ for all x. Therefore, $A = \frac{1}{1+\frac{G''(1)}{G'(1)}1}$ for every relative price level (for all x) Since $\widetilde{\theta}(1) = -\frac{G'(1)}{1\times G''(1)} = \theta$ is the steady state level of elasticity of substitution, we can write $A = \frac{1}{1-\frac{1}{\widetilde{\theta}(1)}} = \frac{\theta}{\theta-1}$, thereby generating constant markups. However, for the general case, the coefficient A changes as the relative price changes and hence markup is not constant at the steady state level.

3.4 Solution

We solve the model by log-linearizing it around a zero inflation steady state. At this steady state equilibrium, RER is set to 1 due to the common preferences assumption. Furthermore, the stochastic discount factor $Q_{t,t+s}$ is set equal to discount factor β^s. The symmetry of the countries result in equal prices of all intermediate firms, levels of employment, and allocations of consumption, imports, and exports for both countries in the steady state.

From now on, all the lower case variables refer to log deviations from the steady state. The complete set of linearized equations are presented in Appendix A. As an example, equation (55) gives the linearized version of the price setting equation (equation 32) for home goods that are sold at home in home currency. The other price setting equations are analogues and can be found in Appendix A.

$$x_{H,k,t} = (1-\beta(1-\alpha_k))(p_t + mc_{t,k}) + \beta(1-\alpha_k)E_t x_{H,k,t+1} \tag{55}$$

where, real marginal cost[9] is linearized as:

$$mc_{k,t} = \sigma c_t + \gamma l_t \tag{56}$$

$$+\frac{1-\chi}{\chi}\begin{bmatrix} \omega(y_t - \theta(x_{H,k,t} - p_{H,k,t}) - \rho(p_{H,k,t} - p_{k,t}) - \eta(p_{k,t} - p_t)) \\ +(1-\omega)(y_t^* - \theta(x_{H,k,t} - q_t - p_t + p_t^* - p_{H,k,t}^*) - \rho(p_{H,k,t}^* - p_{k,t}^*) - \eta(p_{k,t}^* - p_t^*)) \end{bmatrix}$$

[9]Here, we defined the marginal cost at its most general form for all $0 < \chi \leq 1$. Under our baselie CRS setup, linearized real marginal cost is reduced to $\sigma c_t + \gamma l_t$.

Note that here we plugged the real wage from the household's maximization problem: $w_t - p_t = \sigma c_t + \gamma l_t$.

Decreasing returns to scale assumption might potentially end up in firm specific marginal costs. However, we make a simplifying assumption that each sector firms that reset their prices face the same conditional distribution for all future variables that matter for price setting, including marginal costs. Thus we follow Carvalho and Nechio (2011) and simplify the notation using a price that is common to all adjusting firms, and also a common marginal cost.

We follow Eichenbaum and Fischer (2004) and Smets and Wouters (2007) for the linearization of the variable markup model. They show in detail that the additional multiplier in front of the marginal cost, A, will be reduced to the following form once the price setting equation is log linearized around the relevant steady state:

$$A = \frac{1 + G''(1)/G'(1)}{2 + G'''(1)/G''(1)}$$

If we define elasticity of substitution as: $\widetilde{\theta}(x) = -\frac{G'(x)}{xG''(x)}$ and the super elasticity as: $\widetilde{\epsilon}(x) = \frac{\widetilde{P}}{\widetilde{\theta}(1)} \frac{\partial \widetilde{\theta}(1)}{\partial \widetilde{P}}$, where $\widetilde{P}$ is the relative price of the firm, Eichenbaum and Fischer (2004) show that $A = \frac{1}{1+(\zeta-1)\epsilon}$, where $\zeta = \frac{\widetilde{\theta}(1)}{\widetilde{\theta}(1)-1}$ is the steady state level of markup. Gopinath and Itskhoki (2010a) rewrite this coefficient as $A = \frac{1}{1+\Gamma}$, where $\Gamma = \frac{\epsilon}{\theta - 1}$ is the *markup elasticity.*

In the end, the existence of variable markups adds another multiplier in front of the marginal cost. To be more specific, the price setting equation in this case can be written as:[10]

$$x_{H,k,t} - p_{H,k,t-1} - \beta(1-\alpha_k) E_t x_{H,k,t+1} = (1-\beta(1-\alpha_k)) \frac{1}{1+\Gamma} (p_t + mc_{t,k}) + (p_{H,k,t} - p_{H,k,t-1}) \tag{57}$$

Under CES, $\Gamma = 0$ and thus, $A = \frac{1}{1+\Gamma} = 1$. For the general case, on the other hand, $A < 1$. This introduces additional persistence into our model. As before, firms cannot change their prices as much as they want since they have to wait until Calvo fairy touches them. This is the source of nominal rigidity in the model. On the other hand, they are not as willing as before to change their price since markup is a decreasing function of their relative price. This is the

[10] Here, we slightly used a different but common version of equation 32 following Gopinath and Itskhoki (2010a) for technical reasons they explain. They state that this version ensures stationarity of the right hand side. However, both equations (32) and (58) give the same outcome when $\Gamma = 0$, and hence this a non-trivial change.

source of real rigidity in the model. The smaller A from 1 (or equivalently the larger markup elasticity Γ) the higher the degree of strategic complementarity and hence persistence are.

At this point, it would be useful to examine the implications of introducing variable markups into the model for both CRS and DRS cases in more detail. When the production technology is CRS ($\chi = 1$), equation (58) can be written as:

$$x_{H,k,t}-p_{H,k,t-1}-\beta(1-\alpha_k)E_t x_{H,k,t+1} = (1-\beta(1-\alpha_k))A(p_t+\sigma c_t+\gamma l_t)+(p_{H,k,t}-p_{H,k,t-1})$$

where $A = \frac{1}{1+\Gamma}$. Thus, when $\Gamma > 0$, increases in wage rate induce smaller increase in the firm's price, compared to the constant markup case. Chari, Kehoe and McGrattan (2000) term this as *wage effect*. Therefore, wage effect by itself makes the price less sensitive to changes in demand/output.

On the other hand, when the production technology is DRS ($\chi < 1$), the exact multiplier after plugging equation (56) into equation (58) and rearranging, will be $A\Xi$ rather than A only, where $\Xi = \frac{1}{1+A\frac{(1-\chi)}{\chi}\theta}$. Note that as Γ increases, A gets smaller and hence Ξ gets larger ($\Xi = 1$, when $\chi = 1$). This second effect is termed as *output effect* by Chari, Kehoe and McGrattan (2000). This effect alone says that increases in economy wide output increase the firm's marginal cost due to decreasing returns to scale. This, in turn, will increase the incentives of the firms to charge a higher price. Thus, when we have a higher markup elasticity, the multiplier declines more in CRS case than DRS case and hence the effect of the change in markup elasticity would be lower under DRS, since both effects will work in different directions.

Both Klenow and Willis (2006) and Gopinath and Itskhoki (2010b) make the assumption that $D_{H,k,t}$ (and $D_{F,k,t}$, $D^*_{H,k,t}$ and $D^*_{F,k,t}$) equals to its steady state value: $\overline{D} = \frac{\theta-1}{\theta}$. Gopinath and Itskhoki (2010b) also provide a proof to the lemma that the first order deviation of $D_{H,k,t}$ from $\overline{D}$ is nil. Given this, the log-linearized equation for the demand for intermediate good ($Y_{H,k,j,t} = \varphi(\frac{P_{H,k,j,t}}{P_{H,k,t}}D_{H,k,t})Y_{H,kt}$) is reduced to $y_{H,k,j,t} = y_{H,k,t} - \theta(d_{H,k,t} + p_{H,k,j,t} - p_{H,k,t}) = y_{H,k,t} - \theta(p_{H,k,j,t} - p_{H,k,t})$, which is same as the CES case. Thus, in log-linear form, the demand for intermediate varieties can be approximated by our linear equation in the baseline model. Finally, following Gopinath and Itskhoki (2010a), the price levels in sectors can be written same as before: $p_{H,k,t} = \alpha_k E_j x_{H,k,t} + (1-\alpha_k)p_{H,k,t-1}$. As usual, the other sectoral price indices are analogous.

3.5 One Sector Economy

In order to see the effects of heterogeneity in terms of generating aggregate RER persistence, we create a counterfactual one sector economy like that in Carvalho and Nechio (2011) to employ as a basis of comparison. In this economy, the probability of re-setting the price, $\overline{\alpha}$, is set equal to the weighted average of frequency of adjustment of all sectors in the multi-sector economy: $\overline{\alpha} = \sum_{k=1}^{K} f_k \alpha_k$. We also replace the weighted average $\overline{\alpha}$, with the median of the distribution α_{Med} to see if using median instead makes any difference for our results.

3.6 A Three-Sector Approximating Economy

Different from Carvalho and Nechio (2011), we also create a three-sector *approximating* economy. In this model, we have only three sectors rather than 67 in baseline. Frequency of price adjustment and sectoral weights are calibrated to match the first and second moments of distribution of frequencies of price adjustment and average duration of spell in the multisector model. The aim here is to show that the results from multisector model can be approximated reasonably well by using a three-sector approximation model, thereby enabling us to save big time in computation.

4 PARAMETERIZATION

The world consists of two economies that can be interpreted as US and the rest of the world. A period of the model is one month and parameters are calibrated accordingly. The heterogeneity is introduced by the differences of sectors in their frequency of price adjustment. We will use the frequencies and the related weights for 67 sectors that are aggregated by Carvalho and Nechio (2011) from the 272 sectors data of Nakamura and Steinsson (2008). The details of the data and the issues related to aggregating can be found in these mentioned papers. When the weighted average of sectoral frequency of price adjustment is calculated, we obtain the average monthly frequency of price adjustment of the one sector economy: $\overline{\alpha} = 0.2112$. This is equivalent to saying that firms change their prices once in every $1/.2112 = 4.7$ months. Also, the distribution of frequency of price adjustment has a median value of 0.0846.

Our baseline calibration mainly follows Kehoe and Midrigan (2007). In this framework, we assume log utility. Hence, we set $\sigma = 1$ in the baseline. However, we also consider the case $\sigma = 3$ to compare the results to that of Carvalho and

Nechio (2011). We also assume unit Frisch elasticity of supply of labor: $\gamma = 1$. For the production side, we again take the model of Kehoe and Midrigan (2007) as baseline and set $\chi = 1$. In other words, production technology is linear in labor and since the labor is the only factor of production, our baseline model is CRS. We also repeat the exercises for the calibration of Carvalho and Nechio (2011), in which $\chi = 2/3$. This implies "decreasing returns to scale" production function since we do not have capital or any other factor other than labor. This is the only source of real rigidity and strategic complementarity in the baseline model of Carvalho and Nechio.(2011).[11]

Rest of the parameters are from the calibration of Carvalho and Nechio (2011). The discount factor β is set equal to 0.9983 to match 2 percent discount rate per year. The elasticity of substitution across the varieties within a sector θ is set equal to 10. This choice implies a markup rate of 11%, consistent with data. The elasticity of substitution between home and foreign goods ρ is set to 1.5 and the elasticity of substitution across sectors η is set to 1. Finally, share of imports in final good production is set equal to 10%.

Regarding the exogenous aggregate demand persistence, we mainly focus on two values: $\rho_z \in \{0.35, 0.8\}$ even though we check the persistence at every level of ρ_z between 0 and 0.9. Here, 0.8 is chosen as it is consistent with monthly aggregate nominal persistence in data, which falls between 0.75 and 0.9 (Mankiw and Reis, 2002). 0.35, on the other hand, is chosen to match the properties of the nominal exchange rate as it in Imbs, Mumtaz, Ravn and Ray (2005). Our aim is to generate a reasonable level of aggregate RER persistence at this level of nominal aggregate persistence. The standard deviation of the shocks σ_ε is set to $\dot{0}.05/\sqrt{3}$, which roughly matches 1% at quarterly frequency as in Imbs, Mumtaz, Ravn and Ray (2005).

3-sector model is calibrated to approximate the 67-sector model in order to examine whether we can obtain similar results with considerably less amount of computation time. To calibrate the sectoral weights and frequencies of price adjustment, we match the first and second moments of distribution of frequency of adjustment in 67 sectors, as well as the first moment of the average duration of spell. The resulting parameters are depicted in the last two rows of Table 1.

[11] Note that when we set $\chi = 1$, $\gamma = 0$ and $\sigma = 1$, the model becomes strategically neutral.

Table 1. Parameters of the Baseline Model	
Description	Value
All Models	
Discount factor	$\beta = 0.9983$
Coefficient of relative risk aversion	$\sigma = \{1, 3\}$
Elasticity of Labor Supply	$\gamma = 1$
Production technology	$\chi \in \{2/3, 1\}$
Elasticity of substitution across varieties	$\theta = 10$
Elasticity of substitution between home and foreign goods	$\rho = 1.5$
Elasticity of substitution across sectors	$\eta = 1$
Share of imports	$\omega = 0.9$
Aggregate persistence	$\rho_z \in \{0.35, 0.8\}$
Standard deviation of the shock	0.0029
1-Sector Model	
Average frequency of price adjustment	$\overline{\alpha} = 0.2112$
Median of frequency of price adjustment	$\alpha_{Med} = 0.0846$
3-Sector Approximation Model	
Frequencies of price adjustment	$\alpha = \begin{bmatrix} 0.6116 \\ 0.1225 \\ 0.0323 \end{bmatrix}$
Sectoral weights	$f = \begin{bmatrix} 0.2235 \\ 0.5484 \\ 0.2281 \end{bmatrix}$

Following Gopinath and Itskhoki (2010a and 2010b), we will analyze different levels of markup elasticity: $\Gamma \in \{0, 1, 1.5, 4, 10\}$. The values of ϵ that give these values, given $\theta = 10$, are $\{0, 9, 13.5, 36, 90\}$. Here, $\epsilon = 0$ refers to the CES case, while $\Gamma = 1.5$ ($\epsilon = 13.5$) is the markup elasticity estimated by Gopinath and Itskhoki (2010a) for wholesale sector. $\Gamma = 1$, on the other hand, matches the middle of the long run exchange rate pass through range in Gopinath and Itskhoki (2010b). Finally, we use $\Gamma = 10$ to see the effect of very high levels of real rigidities.

5 QUANTITATIVE FINDINGS

In this section, we first describe the measures that we will use as indicators of aggregate RER persistence. Later, the quantitative results from the baseline 1-sector, 3-sector and 67-sector models will be discussed using these measures. Finally, the results under variable markups will be presented for both the CRS and DRS cases.

5.1 Measures of Persistence

We will employ five different measures of persistence, all of which are directly calculated using the Impulse Response Functions (IRFs): hL, uL, qL, CIR and ρ_1. Among these, hL is the one that we will focus on the most, in consistence with the literature. hL refers to the *half life* of the shocks. Half life is defined and calculated pretty standard. It measures the number of periods it takes for the IRFs to a unit shock to dissipate by half. Upper life (uL) and quarter life (qL) are two other measures provided by Steinsson (2008). Upper life refers to the number of periods for IRFs to fall below the initial response while quarter life is the time it takes to drop below one fourth of it. If the IRFs are not hump-shaped and hence always declining, then the uL is said to be equal to 0. These measures are provided to give a better picture of various properties of IRFs.

Cumulative Impulse Responses (CIR) measures the total magnitude of response, normalized by the initial response. In general, the higher the CIR, the more persistent a shock is. To calculate CIR, we divide every period's impulse response by initial response and sum them up. Finally, we also calculate ρ_1, which is the first order autocorrelation observed in IRFs.

5.2 Constant Markups

The results from the baseline model with CRS production technology are presented in Table 1. Note that, we set $\rho_z = 0.8$ as in Carvalho and Nechio (2011) in this subsection. Table 1 shows that the counterfactual 1-sector economy with frequency of price adjustment set to weighted average is too far from generating an empirically realistic level of RER persistence. hL in 1-sector model is only 14 months, which is almost one-third of even the lowest limit of RER persistence seen in data.[12] The results are very similar to other measures of persistence as well. When we set the frequency of price adjustment at median instead, the persistence almost doubles, while still being considerably below the data values. On the other hand, introduction of heterogeneity improves the results considerably, bringing hL to a level that is within the limits found empirically. To be more specific, 67-sector model generates a hL of 40 months, which falls between $3-5$ years range consensus view. Furthermore, uL is 26 months, which is very close to data value. Nevertheless, the 67-sector does poorest in terms of generating a high enough qL measure consistent with data, albeit the significant improvement compared to the counterfactual 1-sector economy. Finally, the measures CIR and ρ_1 also show that heterogeneity increases aggregate RER persistence. Overall, the heterogenous multisector model increase the persistence of RER significantly, compared to the 1-sector model (regardless of using

[12] See Rogoff (1996) and Steinsson (2008) for the measures in data.

mean or median), thereby providing a better account for RER dynamics. As a consequence, we can say that the effect of heterogeneity on explaining RER persistence is quite large.

Table 1. RER Persistence Under Baseline Specification
(CRS, $\rho_z = 0.8$)

	Data	*1-Sector (Mean)*	*1-Sector (Median)*	*67-Sector*
hL	$36-60$	14	28	40
uL	28	9	19	26
qL	76	18	35	54
CIR	–	20.5	49.8	69.9
ρ_1	–	0.969	0.988	0.991

The results from the calibration of Carvalho and Nechio (2011) are almost the same (Table B.1 in Appendix B). Remember that their model has a DRS production technology, along with $\sigma = 3$ preferences. Similar to our CRS case, 1-sector model has a hL of only 14 months, while it increases to 39 months for the multisector model with heterogeneity in frequency of price adjustment. The rest of the statistics are also very similar as well, other than a slightly higher right skewness for multisector model and hence the main message remains the same: heterogeneity in frequency of price adjustment brings RER dynamics to levels consistent with data, explaining most of the PPP puzzle. Hence, we can say that our baseline calibration and that of Carvalho and Nechio (2011) do almost equally well in explaining the aggregate RER persistence.

The findings in Table 1. reveals that the improvement in aggregate RER persistence is not due to only one specific portion of the IRFs. This is further confirmed in Figure 1.[13], where we can see that the overall response of multi-sector model is larger and more persistent at every period after a 1 standard deviation shock to home nominal aggregate demand (which can be thought of as a shock to monetary policy as well). Even though the initial responses are more or less the same in both 1-sector and 67-sector economies, the response of the latter always remains well above that of the former.

[13] From now on, including Figure 1, 1-Sector always refers to the counterfactual one sector economy for which the frequency of price adjustment is set at weighted average of that of 67 sectors (not median).

Figure 1. Scaled IRFs of Aggregate RER to 1 Standard Deviation Shock to Nominal Aggregate Demand[14][15]
(CRS, $\rho_z = 0.8$)

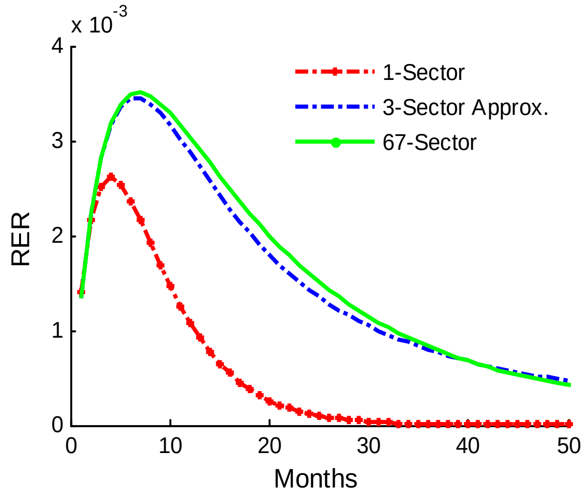


Figure 1 reveals one other crucial result. When we match the first and second moments of frequency of price adjustment and average duration of spell, we can approximate the 67-Sector economy reasonably well with a representative 3-Sector economy. This saves big time in computation and hence increase efficiency, without causing any loss in the main messages of this paper. Figure B.1 in Appendix B shows that the IRFs are almost identical for the calibration of Carvalho and Nechio (2011).

Even though we were able to generate realistic levels of aggregate RER persistence by introducing heterogeneity in frequency of price adjustment, this result relies on a high level of exogenous persistence: $\rho_z = 0.8$. This level of exogenous persistence is consistent with the evidence on the dynamic properties of nominal aggregate demand and it is very common in the literature. However, as Imbs, Mumtaz, Ravn and Ray (2005) show and Carvalho and Nechio (2011) replicate, this high level of exogenous persistence is not consistent with the dynamic properties of nominal exchange rates. Indeed, if we were to calibrate ρ_z to match the properties of nominal exchange rate, it should have been around 0.35. Hence, we are producing realistic aggregate RER persistence at the expense of producing counterfactual implications for the dynamic properties of the nominal exchange rate.

[14] IRFs shows the deviations from the steady state.

[15] IRFs for 1-Sector model is multiplied by 1.3 to equalize the initial responses in both 1-Sector and 67-Sector economies. The initial responses are not too different but they are equalized to see the overall picture better.

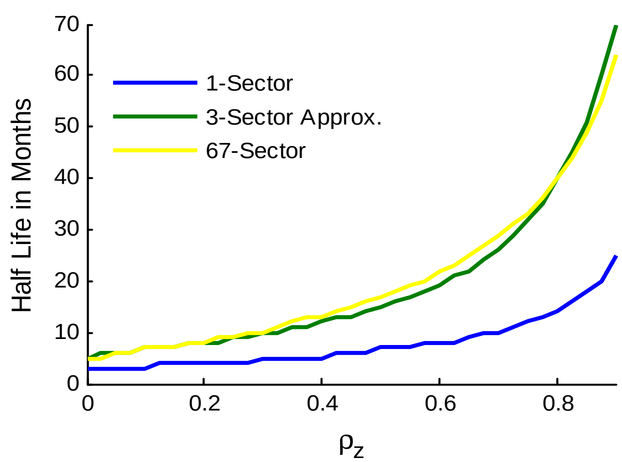
At his point, a natural question emerges whether our model is capable of producing a realistically high degree of aggregate RER persistence when the nominal aggregate demand persistence is set to a level consistent with nominal exchange rate dynamics. To answer this question, we repeat the same exercises, this time setting $\rho_z = 0.35$ (Table 2). Also, we run the model for different values of ρ_z, to see the relationship between the exogenous nominal demand persistence and RER persistence (Figure 2).

Table 2. RER Persistence Under Lower Exogenous Persistence
(CRS, $\rho_z = 0.35$)

	Data	*1-Sector*	*67-Sector*
hL	$36 - 60$	5	12
uL	28	1	3
qL	76	8	23
CIR	$-$	6.2	18.7
ρ_1	$-$	0.872	0.948

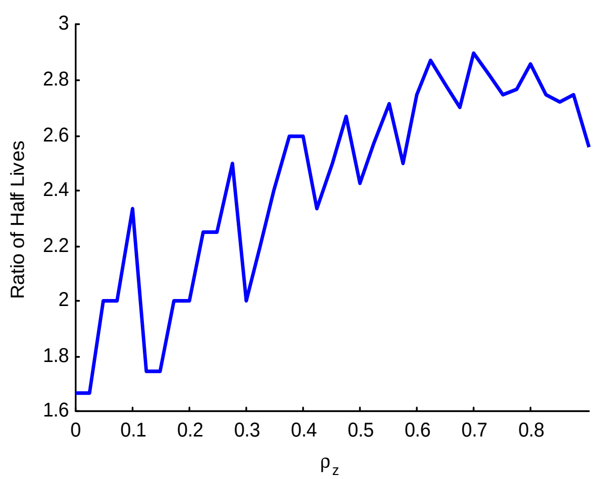
Table 2. shows that both the 1-Sector and multisector models do very poorly in terms of generating a realistic aggregate RER persistence, even though heterogeneity still brings the results closer to the levels somehow less inconsistent with data. The half-life of IRF after a shock to nominal aggregate demand is only 1.year in the multisector model, which is very low compared to $3 - 5$ years in data. The model does poorly in replicating the other dynamics that can be measured by uL and qL as well. This table is reproduced for the DRS case of Carvalho and Nechio (2011) and the results are very similar for the 1-Sector model, while 67-Sector model does poorer under DRS case than our CRS case (Table B.2., Appendix B).

Figure 2. Half-Life as a Function of Aggregate Persistence
(CRS)



When we repeat the same exercise for a range of values of ρ_z between $[0, 0.9]$, we obtain four main results about the interaction of exogenous aggregate persistence, RER persistence and heterogeneity in frequency of price adjustment: (i) For all levels of exogenous persistence, the heterogenous multisector does better than the counterfactual 1-Sector economy in terms of generating a higher level of RER persistence, (ii) 1-Sector economy is not capable of producing realistic levels of RER persistence even under extremely high levels of nominal aggregate demand persistence, while the multisector economy is capable of generating realistic levels of aggregate RER persistence only with high levels of nominal aggregate demand persistence, (iii) unlike Carvalho and Nechio (2011) who claims that the amplification ratio, which is defined as the ratio of half life of the multisector model to that of 1-Sector model, is strictly increasing in aggregate persistence, our results for both CRS case and DRS case with their calibration reveals that "strictly increasing" is a very strong comment (Figure 3). There are some ups and downs in the amplification ratios as ρ_z increases. Still, we agree with them on that this ratio has an increasing tendency generally. Hence, we can conclude that, the contribution of heterogeneity in terms of generating higher RER persistence increases with nominal aggregate demand persistence. (iv) Once again, 3-Sector artificial model approximates the 67-Sector model reasonably well for every level of exogenous persistence. Figures B.2. and B.3 in Appendix B reproduce the same results for the DRS case.

Figure 3. Amplification Ratio between Multisector and 1-Sector Models[16]
(CRS)



In summary, our model's success in explaining RER dynamics totally depends on the existence of a very high level of aggregate persistence. Carvalho

[16] Here, we divide half life of 67-Sector by that of 1-Sector for each level of ρ_z.

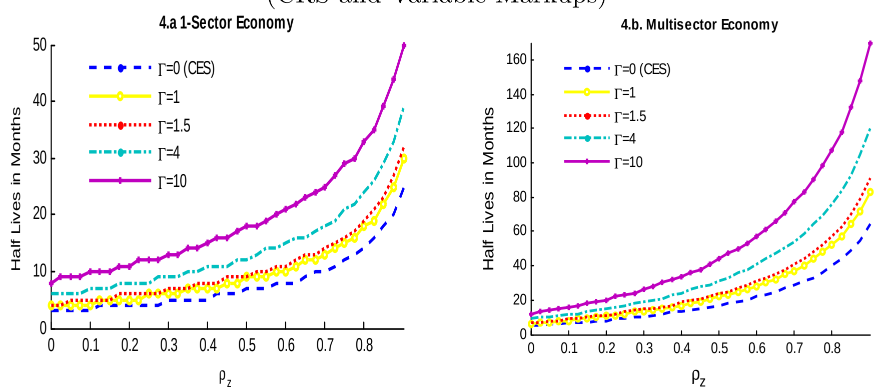
and Nechio (2011) explain this by the lack of real rigidities in the model (other than the DRS in their baseline calibration). To show that real rigidities might improve the RER persistence at low levels of nominal aggregate demand persistence, they increase both the elasticity of substitution across varieties and elasticity of substitution across sectors to unrealistically high levels.[17] They show that, with this calibration, the heterogeneous multisector model is able to generate realistic aggregate RER persistence even at $\rho_z = 0.35$. However, once again, they do this by only counterfactually high level of elasticities and make an inference from that calibration about the results of having real rigidities in the model. Our aim is to introduce real rigidities through variable markups to test their prediction against Kehoe and Midrigan (2007) and Chari, Kehoe and McGrattan (2000), among various others, who claim that real rigidities do not improve persistence significantly in the absence of high aggregate persistence. When doing so, we will keep the parameters at their empirically consistent levels and see if prediction of Carvalho and Nechio (2011) is verified under plausible parameterization.

5.3 Variable Markups

CRS Calibration:

Figure 4. Half-Life as a Function of Aggregate Persistence and Markup Elasticity

(CRS and Variable Markups)



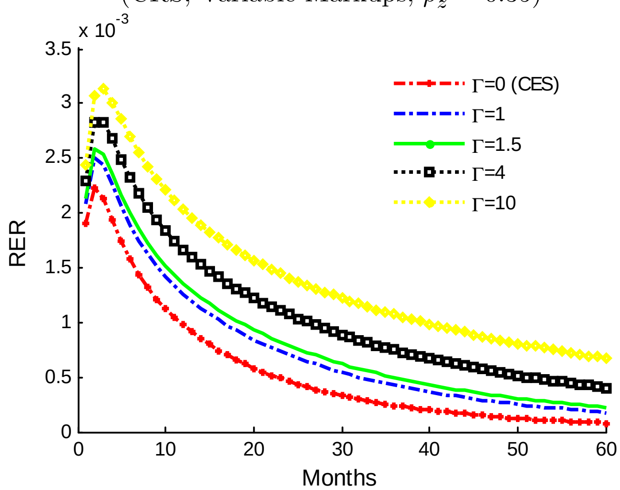
First of all, as expected, incorporation of variable markup channel of real rigidities into the model increases the aggregate RER persistence for both 1-Sector and 67-Sector economies at every level of nominal aggregate demand persistence (Figures 4a and 4b). The question here is whether realistic aggregate

[17] They set $\theta = 30$ and $\eta = 30$.

RER persistence is generated when $\rho_z = 0.35$ and markup elasticity is set at empirically supported levels such as $\Gamma = 1$ or $\Gamma = 1.5$. The answer appears to be *"no"* when Figures 4 and 5 and Table 3 are examined. The half life of IRFs to a 1 standard deviation shock to nominal aggregate demand increases from 12 months to only 16 months, when $\Gamma = 1.5$. The improvement is at relatively similar magnitudes for other measures as well.

Figure 5. IRFs of Aggregate RER to 1 Standard Deviation Shock to Nominal Aggregate Demand

(CRS, Variable Markups, $\rho_z = 0.35$)



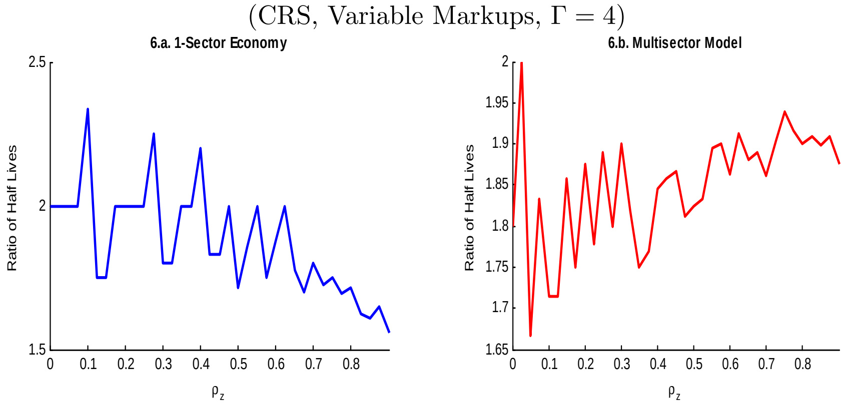
We can generate aggregate RER persistence close to what is observed in data, only when markup elasticity is unrealistically large. Even when $\Gamma = 10$, the half life increases from 12 months under CES case to 30 months, still falling short of 36 months, which is the lower bound of empirically plausible half life measure. On the other hand, counterfactual 1-Sector economy falls significantly short of data in terms of generating high aggregate RER persistence, even under our extremely high markup elasticity: $\Gamma = 10$.

Table 3. RER Persistence Under Lower Exogenous Persistence
(CRS, Variable Markups, $\rho_z = 0.35$)

		hL	**uL**	**qL**	**CIR**	ρ_1
Data		$36-60$	28	76	–	–
$\Gamma = 0$	*1-Sector*	5	1	8	6.2	0.872
(CES)	*67-Sector*	12	3	23	18.7	0.948
$\Gamma = 1$	*1-Sector*	7	2	10	8.5	0.907
	67-Sector	15	3	31	24.9	0.960
$\Gamma = 1.5$	*1-Sector*	7	2	11	9.3	0.916
	67-Sector	16	4	34	27.3	0.963
$\Gamma = 4$	*1-Sector*	10	3	15	12.8	0.940
	67-Sector	21	5	45	36.5	0.972
$\Gamma = 10$	*1-Sector*	14	5	22	18.7	0.959
	67-Sector	30	6	65	50.4	0.980

When we calculate the amplification ratio between the baseline CES case and the model with markup elasticity of $\Gamma = 4$ for all values of ρ_z, several interesting results emerge: (i) the contribution of variable markups are larger on average for 67-Sector economy than their influence on counterfactual 1-Sector economy and (ii) more surprisingly, the contribution of variable markups in terms of increasing the RER persistence is decreasing in ρ_z for 1-Sector economy and increasing in ρ_z for 67-Sector economy (Figure 6).

Figure 6. Ratio of Half-Lives of Multisector Model and 1-Sector Model[18]
(CRS, Variable Markups, $\Gamma = 4$)



[18] Here, we divide half life when $\Gamma = 4$ by that when $\Gamma = 0$ (CES) for all levels of ρ_z.

Despite the failure of variable markups channel of real rigidities in generating a realistic level of aggregate RER persistence at $\rho_z = 0.35$, the model with variable markups is still a better replication of data on RER dynamics. For instance, when $\rho_z = 0.8$ as before and markup elasticity Γ is set to 1.5 as in Gopinath and Itskhoki (2010a), the half life generated by the 67-Sector economy increases from 40 months to 57 months, still remaining within the realistic limits in data. Furthermore, upper life and quarter life increase to 36 and 80 months, which are clearly more consistent with data than the our baseline CES model. Thus, we can say that incorporation of variable markups to the baseline model is better in explaining the aggregate RER dynamics.

Going to back to our main motivation, which is to reduce the required level of aggregate persistence to generate reasonable RER dynamics, we can still see that variable markups makes an improvement in this aspect. When we target 36 months half life and calculate ρ_z that generates this, we can see that the required aggregate persistence declines from 0.77 in baseline model with CES demand to 0.67 for the variable markup model where markup elasticity is set to 1.5 (Table 4).The required level of aggregate persistence declines even further to 0.55 and 0.43 when $\Gamma = 4$ and $\Gamma = 10$ respectively.

Table 4. Required Aggregate Persistence[19]

(CRS)

	$\Gamma = 0$ (CES)	$\Gamma = 1$	$\Gamma = 1.5$	$\Gamma = 4$	$\Gamma = 10$
Required ρ_z	0.77	0.69	0.66	0.55	0.43

Summary of findings:

(i) The effect of heterogeneity in frequency of price adjustment on the aggregate RER persistence is remarkably large. The heterogenous multisector model amplifies the aggregate RER persistence compared to a counterfactual 1-Sector economy. This is true regardless of the level of aggregate persistence. However, the amplification of RER persistence is higher for the large degrees of nominal aggregate demand persistence.

(ii) Confirming Nakamura and Steinsson (2010), if one sector economy is calibrated to have frequency of price adjustment of the median (rather than the weighted average) of the multisector model, the RER persistence improves considerably. However, different from Nakamura and Steinsson (2010), it still does significantly worse than the multisector model in terms of matching RER data.

(iii) Introducing variable markups through Kimball (1995) demand adds one other channel of persistence since firms will be less willing to change their prices,

[19] Here, aggregate persistence level ρ_z required to generate half life of 36 month is calculated for various levels of markup elasticity Γ.

unless their opponents do so. Hence, the aggregate RER persistence is larger in a model with variable markups than a model with constant markups.

(iv) The amplification effect of variable markups remains very modest when the markup elasticity is set at an empirically supported level. The absolute effect of variable markups gets less significant as exogenous nominal aggregate demand persistence gets smaller.

(v) Some degree of aggregate demand persistence is required for explaining the RER dynamics realistically. Our model is capable of generating RER persistence consistent with the empirically found degree only when markup elasticity is set at implausibly high levels (which corresponds to unrealistically high levels of superelasticity and thus, too large convexity of demand function).

(vi) Despite its inability to fully reconcile the dynamics of nominal and real exchange rates, the model with variable markups with a markup elasticity set at empirically supported levels closes some of the gap between theory and data. Hence, we can say that the model with variable markups is more successful in explaining the RER dynamics than our baseline model with CES demand since it relies on a smaller level of aggregate persistence. Furthermore, this model generates measures that are more consistent with RER dynamics in data, compared to the model with constant markups.

(vii) All these show that adding variable markups to the baseline model was a right but insufficient direction of research. Thus, other sources of real rigidities to increase endogenous persistence should be sought. At this point, the first candidate would be introducing sector specific capital. However, in a recent ongoing paper, Carvalho and Nechio (2012) nest sector specific capital to their model and surprisingly find that even though incorporating capital into the baseline model increases RER persistence, making it sector specific reduces the RER persistence. Similarly, in an extension to their model, Kehoe and Midrigan (2007) change their basic setup and allow the intermediate firms to use the intermediate goods as inputs. They also find that this form of real rigidity does not increase the RER persistence.

DRS Calibration:

We repeat the same analysis for the calibration of Carvalho and Nechio (2011) in order to compare our results with their prediction about incorporation of real rigidities based on their numerical exercise.[20] Figure 7 shows that the effect of variable markups are even less significant than under our CRS calibration.[21] Especially, when markup elasticity is set at empirically supported

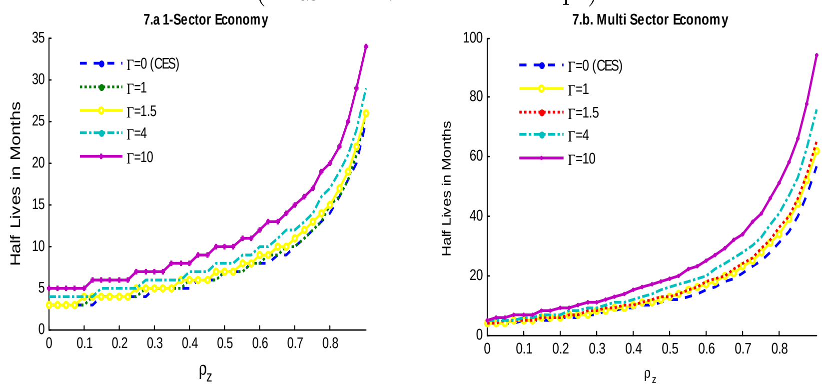
[20] Remember that $\chi = 2/3$, $\gamma = 1$ and $\sigma = 3$ in their baseline model.

[21] Remember that both CRS and DRS cases generate RER persistence at around same levels under the constant markup case. However, when variable markups are introduced, two models differ significantly. As explained in Section 3, when markup elasticity Γ increases, firms will be less willing to raise their price since the higher their relative price, the lower the demand will be. This corresponds to "wage effect" as termed by Chari, Kehoe and McGrattan (2000) and it is common for both CRS and DRS cases. On the other hand, when there is decreasing returns to scale, there also exists a second channel: "output effect", which induces the monopolist to raise the price as explained in Section 3. When markup elasticity increases,

levels such as 1 and 1.5, the increase in half lives are negligibly small, ranging only between 0 and 2 months for the 1-Sector economy and between 1 and 8 months for the multisector economy.[22]

Figure 7. Half-Life as a Function of Aggregate Persistence and Markup Elasticity

(DRS and Variable Markups)



Why are our findings significantly different from the predictions of Carvalho and Nechio (2011)? First of all, in their analysis to predict the effects of introducing real rigidities, they set the elasticities of substitution across varieties and sectors at unrealistically large levels: $\theta = 30$ and $\eta = 30$. More important than this, they change their calibration from the baseline and set $\gamma = 0$ (linear utility in labor) and $\sigma = 1$. However, comparing the results from this alternative parameterization to the results from baseline calibration is fallacious. The model already generates higher RER persistence with this calibration (a half life of 67 months rather than 39 months in baseline calibration) and hence the results are not comparable. Indeed, they overstate the likely effects of real rigidities by not reporting the RER dynamics of this alternative calibration when $\theta = 10$ and $\eta = 1$ as in the baseline. Figure 8 shows that the improvement due to adding variable markups would "seem" larger if we had used their alternative calibration and compared it to our baseline calibration. Indeed, even when $\Gamma = 1.5$, the multisector model[23] is able to replicate the RER dynamics at our targeted aggregate persistence: $\rho_z = 0.35$, without the need of increasing the elasticities of substitution to unrealistically high levels as Carvalho and Nechio (2011) did. Hence, our results are not actually contradicting, but clarifying the findings and predictions from Carvalho and Nechio (2011).

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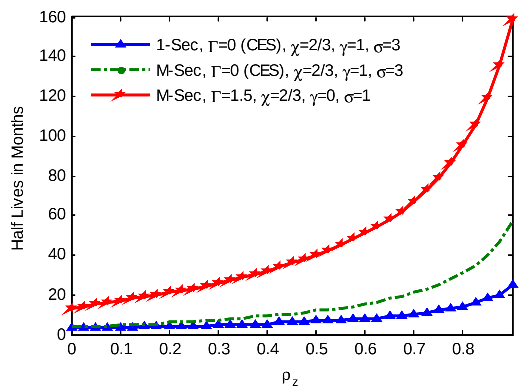
the output effect becomes larger. This is what differentiates CRS and DRS cases. They both are affected the same by the wage effect, while income effect reverses some of the wage affect under DRS.

[22] In this graph, we employed 3-Sector approximating economy, which is already shown to be a reasonable approximation of the 67-Sectors economy.

[23] Here, again, 3-Sector approximation economy is used.

Figure 8. Half-Life as a Function of Aggregate Persistence and Markup Elasticity
(DRS, Alternative Calibration)



6 CONCLUSION

In this paper, we aimed at explaining the PPP puzzle by introducing the variable markup channel of real rigidities into a multisector, sticky price model with heterogeneity in frequency of price adjustment and local currency pricing. To be more specific, we tried to generate realistic aggregate RER persistence, while keeping the degree of nominal aggregate demand persistence at levels consistent with nominal exchange rate dynamics and all other model parameters at plausible levels. To isolate and analyze the effects of variable markups, we compared the findings from two models: we had CES aggregation with no strategic complementarities that resulted in constant markups. Later, we added a second channel of price rigidity through variable markups generated by a non-CES demand function a la Kimball (1995), which further reduced the incentive of firms to reset their prices unless their opponents do the same.

Our baseline findings mainly confirm those of Carvalho and Nechio (2011). We found that the effect of heterogeneity in explaining the PPP puzzle is remarkable. Regardless of the level of aggregate nominal persistence, the aggregate RER persistence is amplified by heterogeneity in frequency of price adjustment, compared to the counterfactual one sector economy. In the baseline, when the nominal aggregate persistence is set to $\rho_z = 0.8$, the model generates a half life of 40 months in response to a nominal shock, which is within the empirical evidence. The other measures of persistence are also reasonably close to data. Nonetheless, even the multisector heterogeneous economy is unable to generate realistic RER dynamics when the nominal aggregate persistence is set to

$\rho_z = 0.35$- a level consistent with the dynamic properties of nominal exchange rates.

There is no doubt that introduction of the variable markups amplifies the aggregate RER persistence at all nominal aggregate demand persistence levels. Especially, when ρ_z is kept at 0.8, the persistence measures get more consistent with data, even with empirically documented levels of a markup elasticity of 1.5. Also, the required level of nominal aggregate persistence to obtain a realistic level of RER persistence decreases when variable markups are introduced. Hence, the model with variable markups is superior to the baseline model with constant markups in explaining the RER dynamics.

Nevertheless, the model with variable markups is incapable of producing persistence measures consistent with data either, when $\rho_z = 0.35$, despite the improvement achieved compared to the model with constant markups. The only way to reconcile the RER dynamics of our model with dynamics of nominal exchange rates is to set the markup elasticity over 10 (or equivalently set the superelasticity over 90). However, this implies a very large curvature of demand function and this is not empirically plausible. The improvement made by variable markups gets even smaller in magnitude once we switch to the baseline calibration of Carvalho and Nechio (2011), with DRS production technology. The reasons behind this are explained in detail in the previous sections.

Overall, our results with variable markups are more consistent with Kehoe and Midrigan (2007) and several other studies, while they seem to conflict with those of Carvalho and Nechio (2011). However, we showed that this seemingly conflicting results are due to different calibration and unrealistic elasticities of substitution used by Carvalho and Nechio (2011) to predict the likely effects of incorporating real rigidities. Finally, we also showed that 67-sector economy can be approximated by a 3-sector representative economy that is calibrated to match the first and second moments of frequency of price adjustment and average duration of spell.

In summary, we can say that model with variable markups is better in explaining the RER dynamics compared to the baseline model, hence it should be preferred. However, we should also look for other forms of real rigidities. At this point, the first candidate would be introducing sector specific capital as it is common in the literature. However, in a recent ongoing paper, Carvalho and Nechio (2012) nested sector specific capital to their model and surprisingly found that even though incorporating capital into the baseline model increases RER persistence, making it sector specific reduces the half life obtained. In an extension to their model, Kehoe and Midrigan (2007) change their basic setup and allows the intermediate firms to use intermediate goods as input to production. They also find that this channel of real rigidities cannot increase the RER persistence. Therefore, we should seek other sources of real rigidities, which will not require a high degree of nominal aggregate persistence to replicate RER dynamics. Apart from real rigidities, one other research topic might be to estimate the markup elasticity for each of those 67 sectors used in this paper and see if this additional heterogeneity improves the results.

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APPENDIX A:[24]

Linearized System of Equations

The system is linearized around a symmetric steady state with zero inflation rate. At this steady state equilibrium, RER is set to 1 due to the common preferences assumption. Furthermore, the stochastic discount factor $Q_{t,t+s}$ is set equal to discount factor β^s, while all the prices have a steady value of 1. The latter is due to the symmetry of the countries, which result in equal prices of all intermediate firms, levels of employment, and allocations of consumption, imports, and exports for both countries in the steady state. Remember that, all the small case letters refer to log-deviations from the steady state.

Household's Problem:

From the problem of the households we have equations (3), (6) and (10), which define real wages and real exchange rate:

$$\begin{aligned} w_t - p_t &= \sigma c_t + \gamma l_t \\ w_t^* - p_t^* &= \sigma c_t^* + \gamma l_t^* \\ q_t &= \sigma (c_t - c_t^*) \end{aligned}$$

Price Equations:[25]

$$\begin{aligned} p_t &= \sum_{k=1}^{K} f_k p_{k,t} \\ p_t^* &= \sum_{k=1}^{K} f_k p_{k,t}^* \\ p_{k,t} &= \omega p_{H,k,t} + (1-\omega) p_{F,k,t} \\ p_{k,t}^* &= (1-\omega) p_{H,k,t}^* + \omega p_{F,k,t}^* \\ p_{H,k,t} &= \alpha_k x_{H,k,t} + (1-\alpha_k) p_{H,k,t-1} \\ p_{F,k,t} &= \alpha_k x_{F,k,t} + (1-\alpha_k) p_{F,k,t-1} \\ p_{H,k,t}^* &= \alpha_k x_{H,k,t}^* + (1-\alpha_k) p_{H,k,t-1}^* \\ p_{F,k,t}^* &= \alpha_k x_{F,k,t}^* + (1-\alpha_k) p_{F,k,t-1}^* \end{aligned}$$

[24] This appendix mainly follows from appendix of Carvalho and Nechio (2011).

[25] The linearization of these equations are pretty standard and hence the details are skipped here.

We also define domestic price indices for home and foreign produced goods that are sold in home country (and same for foreign country):

$$\begin{aligned} p_{H,t} &= \sum_{k=1}^{K} f_k p_{H,k,t} \\ p_{F,t} &= \sum_{k=1}^{K} f_k p_{F,k,t} \\ p^*_{H,t} &= \sum_{k=1}^{K} f_k p^*_{H,k,t} \\ p^*_{F,t} &= \sum_{k=1}^{K} f_k p^*_{F,k,t} \end{aligned}$$

Output level equations:

Output of each individual firm (variety):

$$\begin{aligned} y_{H,k,j,t+s} &= y_{t+s} - \theta[x_{H,k,j,t} - p_{H,k,t+s}] - \rho[p_{H,k,t+s} - p_{k,t+s}] - \eta[p_{k,t+s} - p_{t+s}] \\ y_{F,k,j,t+s} &= y_{t+s} - \theta[x_{F,k,j,t} - p_{F,k,t+s}] - \rho[p_{F,k,t+s} - p_{k,t+s}] - \eta[p_{k,t+s} - p_{t+s}] \\ y^*_{H,k,j,t+s} &= y^*_{t+s} - \theta[x^*_{H,k,j,t} - p^*_{H,k,t+s}] - \rho[p^*_{H,k,t+s} - p^*_{k,t+s}] - \eta[p^*_{k,t} - p^*_{t+s}] \\ y^*_{F,k,j,t+s} &= y^*_{t+s} - \theta[x^*_{F,k,j,t} - p^*_{F,k,t+s}] - \rho[p^*_{F,k,t+s} - p^*_{k,t+s}] - \eta[p^*_{k,t} - p^*_{t+s}] \end{aligned}$$

with the assumption that firms keep the same price at time t until they have the chance of resetting their price at time $t+s$.

Foreign and domestic sectoral outputs:

$$\begin{aligned} y_{H,k,t} &= y_{t+s} - \rho[p_{H,k,t} - p_{k,t}] - \eta[p_{k,t} - p_t] \\ y_{F,k,t} &= y_t - \rho[p_{F,k,t} - p_{k,t}] - \eta[p_{k,t} - p_t] \\ y^*_{H,k,t} &= y^*_t - \rho[p^*_{H,k,t} - p^*_{k,t}] - \eta[p^*_{k,t} - p^*_t] \\ y^*_{F,k,t} &= y^*_t - \rho[p^*_{F,k,t} - p^*_{k,t}] - \eta[p^*_{k,t} - p^*_t] \end{aligned}$$

$$\begin{aligned} y_{k,t} &= \omega y_{H,k,t} + (1-\omega) y_{F,k,t} \\ y^*_{k,t} &= (1-\omega) y^*_{H,k,t} + \omega y^*_{F,k,t} \end{aligned}$$

Final output:

$$y_t = \sum_{k=1}^{K} f_k y_{k,t}$$
$$y_t^* = \sum_{k=1}^{K} f_k y_{k,t}^*$$

We also define GDP as follows:

$$gdp_t = y_t + (1-\omega)\sum_{k=1}^{K} f_k (y_{H,k,t}^* - y_{F,k,t})$$

When we plug $y_{H,k,t}^*$ and $y_{F,k,t}$ from above, in the end we obtain:

$$gdp_t = y_t + (1-\omega)[y_t^* - y_t - \rho\sum_{k=1}^{K} f_k [(p_{H,k,t}^*, -p_t^*) - (p_{F,k,t} - p_t)]]$$

Analogously:

$$gdp_t^* = y_t - (1-\omega)[y_t^* - y_t - \rho\sum_{k=1}^{K} f_k [(p_{H,k,t}^*, -p_t^*) - (p_{F,k,t} - p_t)]]$$

Price setting equations:

Log-linearization of the equations (32)-(35) around the steady state is standard in the literature. In the end, a price resetting firm takes all the future nominal marginal costs into account when deciding its new prices:

$$
\begin{aligned}
x_{H,k,t} &= (1-\beta(1-\alpha_k))E_t\sum_{j=0}^{\infty}\beta^j(1-\alpha_k)^j(p_{t+j}+mc_{k,t+j}) \\
x_{H,k,t} &= (1-\beta(1-\alpha_k))(p_t+mc_{k,t}) \\
&\quad +\beta(1-\alpha_k)(1-\beta(1-\alpha_k))E_t\sum_{j=0}^{\infty}\beta^j(1-\alpha_k)^j(p_{t+1+j}+mc_{k,t+1+j}) \\
x_{H,k,t} &= (1-\beta(1-\alpha_k))(p_t+mc_{k,t})+\beta(1-\alpha_k)E_t x_{H,k,t+1}
\end{aligned}
$$

Similarly, by the same steps:

$$
\begin{aligned}
x_{F,k,t} &= (1-\beta(1-\alpha_k))(p_t+q_t+mc^*_{k,t})+\beta(1-\alpha_k)E_t x_{F,k,t+1} \\
x^*_{H,k,t} &= (1-\beta(1-\alpha_k))(p^*_t-q_t+mc_{k,t})+\beta(1-\alpha_k)E_t x^*_{H,k,t+1} \\
x^*_{F,k,t} &= (1-\beta(1-\alpha_k))(p^*_t+mc^*_{k,t})+\beta(1-\alpha_k)E_t x^*_{F,k,t+1}
\end{aligned}
$$

where real marginal costs are defined as:

$$
\begin{aligned}
mc_{k,j,t} &= w_t-p_t+(1-\chi)l_{k,j,t} \\
mc^*_{k,j,t} &= w^*_t-p^*_t+(1-\chi)l^*_{k,j,t}
\end{aligned}
$$

Production technology for intermediate firms:

$$
\begin{aligned}
\omega y_{H,k,j,t}+(1-\omega)^* y_{H,k,j,t} &= \chi l_{k,j,t} \\
(1-\omega)y_{F,k,j,t}+\omega^* y_{F,k,j,t} &= \chi l^*_{k,j,t}
\end{aligned}
$$

Market clearing conditions:

$$
\begin{aligned}
\omega[y_t-\rho(p_{H,t}-p_t)]+(1-\omega)[y^*_t-\rho(p^*_{H,t}-p^*_t)] &= \chi l_t \\
(1-\omega)[y_t-\rho(p_{F,t}-p_t)]+\omega[y^*_t-\rho(p^*_{F,t}-p^*_t)] &= \chi l^*_t
\end{aligned}
$$

$$\sum_{k=1}^{K} f_k \int_0^1 l_{k,j,t} dj = l_t$$

$$\sum_{k=1}^{K} f_k \int_0^1 l^*_{k,j,t} dj = l^*_t$$

$$\begin{aligned} c_t &= y_t \\ c^*_t &= y^*_t \end{aligned}$$

Aggregate nominal demand:

$$\begin{aligned} z_t &= p_t + y_t \\ z^*_t &= p^*_t + y^*_t \end{aligned}$$

APPENDIX B:
Additional Figures and Tables

Table B.1. RER Persistence Under the Specification of Carvalho and Nechio (2011)
(DRS: $\chi = 2/3$, $\sigma = 3$, $\rho_z = 0.8$)

	Data	*1-Sector (Mean)*	*1-Sector (Median)*	*67-Sector*
hL	$36-60$	14	27	39
uL	28	9	19	23
qL	76	18	35	57
CIR	–	20.4	49.5	67.2
ρ_1	–	0.969	0.988	0.990

Figure B.1. IRFs of Aggregate RER to 1 Standard Deviation Shock to Nominal Aggregate Demand
(DRS: $\chi = 2/3$, $\sigma = 3$, $\rho_z = 0.8$)

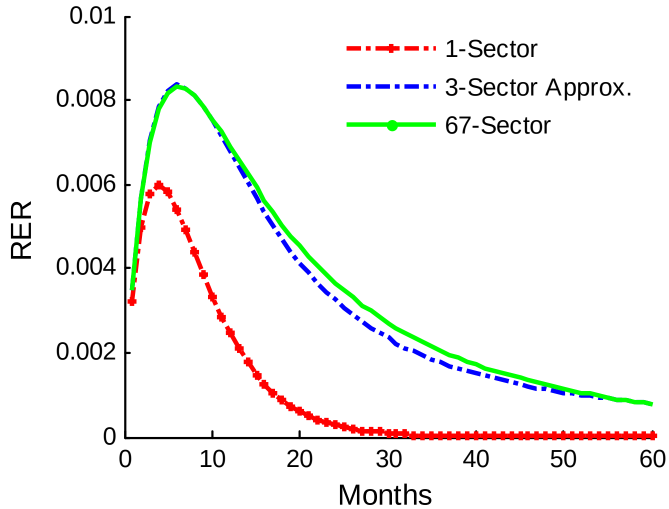

Table B.2. RER Persistence Under Lower Exogenous Persistence
(DRS: $\chi = 2/3$, $\sigma = 3$, $\rho_z = 0.35$)

	Data	*1-Sector (Mean)*	*67-Sector*
hL	$36-60$	5	9
uL	28	1	2
qL	76	8	19
CIR	–	6.2	16.9
ρ_1	–	0.871	0.948

Figure B.2. Half-Life as a Function of Aggregate Persistence
(DRS: $\chi = 2/3$, $\sigma = 3$, $\rho_z = 0.35$)

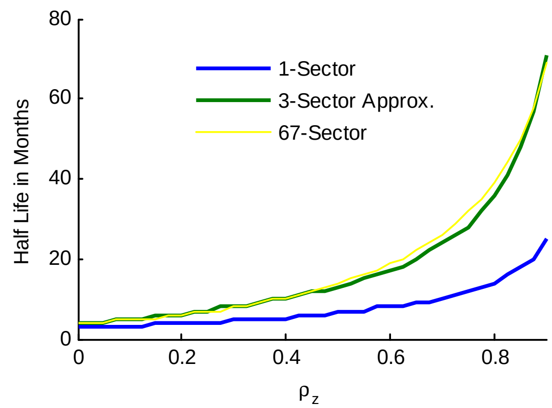

Figure B.3. Ratio of Half-Lives of Multisector Model and 1-Sector Model
(DRS: $\chi = 2/3$, $\sigma = 3$, $\rho_z = 0.35$)