From Job Polarization to Wealth Inequality:
A Quantitative Test of Routinization Hypothesis

MUSA ORAK
Department of Economics, UCLA
e-mail: musaorak@ucla.edu

August 28, 2013

ABSTRACT

A recently growing body of literature has linked the ongoing increase in U.S. wage inequality in last three decades to the job and wage polarization processes. In this paper, we test how much of these employment and wage structure shifts can be explained by the routinization hypothesis alone over the period 1981-2011. We do this in an incomplete markets environment with heterogeneous agents. Incomplete markets and the resulting saving behavior enable us to establish the missing link between job polarization literature and wealth inequality and also to introduce a simple mechanism of educational choice. Our model economy shows that routinization hypothesis alone accounts for a significant portion of the increase in employment share of high skill occupations and their relative wages, as well as the increase in the supply of labor with a college equivalent degree. The model has also been successful in explaining the erosion of wealth in the middle of wealth distribution, thereby indicating the role that might have been played by job and wage polarization on the recently discussed phenomenon of middle class squeeze. On the other hand, the model fails to generate the growth in the share of low skill occupations. Preliminary results show that a two-sector version of the model is remarkably more promising in generating the shifts in both employment and wage structures over the past three decades.

Keywords: Job polarization; Wage polarization; Wage inequality; Wealth inequality; Routinization; Heterogenous agents

JEL Classifications: E21,E24,J24,J31,J62.

1I thank Gary Hansen, Lee Ohanian, Vincenzo Quadrini, Fernando Guiliano, Federico Grinberg, Dennis Kuo and Gabriel Zaurak, as well as the participants of UCLA Macroeconomic Proseminar for helpful and valuable comments and discussions. All the errors are mine.
1 INTRODUCTION

It has been well documented that U.S. wage inequality has been persistently rising in past three decades. There has been an extensive literature trying to explain the reasons behind this trend. A relatively recent literature following (Autor, Levy, and Murnane 2003) and (Autor, Katz, and Kearny 2006) linked the rise in wage inequality to the concepts of job and wage polarization, which have been claimed to characterize the U.S. labor markets beginning from the 1980s. Here, job polarization refers to the growth in the employment share of high skill and low skill occupations, while that of middle skill occupations erodes. Similarly, wage polarization refers to the rise in relative wages of both high and low skill occupations compared to the middle skill ones.

The literature on job and wage polarization has been growing recently. Some studies mainly focus on documenting the existence of job and wage polarization and investigate whether the trend has been unique to U.S. or to some certain period alone. Another line of studies tries to explain the factors leading to the polarization process. Among those, some studies explain job and wage polarization with globalization and offshoring (Goos, Manning, and Salomons 2011; Das 2012; Grossman and Rossi-Hansberg 2008; Costinot and Vogel 2010; Jung and Mercenier 2010; Monte 2011). A more popular explanation, on the other hand, is the "routinization" hypothesis of (Autor, Levy, and Murnane 2003). This paper will focus on the latter and test its predictions in an incomplete markets environment with heterogeneous agents.

As it will be explained in detail in Section 3, the occupations are divided into three categories following (Autor, Levy, and Murnane 2003): abstract, routine and manual. Here, the first group represents high skill occupations while the other two represent middle and low skill occupations respectively. According to the routinization hypothesis, technological change arrives as routine replacement, which we will call as "routinization biased technological change (RBTC)". New technology in this context takes the form of either an exogenous decline in the price of computer capital or an exogenous increase in the productivity of computer capital, which is assumed to be relative substitute with middle skill tasks and relative complement with high skill tasks. Thus, technological improvement, which is introduced as the decline in the price of computer capital, substitutes routine tasks, while it complements abstract tasks. Larger demand for high skill labor following a technological improvement drives the relative
wages of this group up and hence induces an increase in the demand for low
skill service occupations as well. As a consequence, the shares of abstract (high
skill) and manual (low skill) occupations increase.

Most of the studies in the literature are abstracted from market imper-
fections. Due to the lack of market imperfections, job and wage polarization
literature misses the saving behavior and hence the link between this literature
and wealth inequality has not been established yet. Furthermore, other than
the trade models, the literature solely focuses on ‘demand’ side effects on skilled
labor, while ignoring the increase in ‘supply’ of skilled labor that has been ob-
served in data. Therefore, the models usually fail to generate rise in the share of
high skill occupations. In this paper, I will incorporate an otherwise standard
task-based job polarization model into a (Bewley 1977) type heterogenous agents
model with incomplete markets. This will contribute to the existing literature
on job polarization and wage inequality in two ways: first, incomplete markets
will enable us to study the accumulation of savings and hence implications of
job and wage polarization for wealth distribution. Second, having savings in
the model will enable us to introduce a simple mechanism of education choice,
thereby making the supply of skilled labor endogenous. This will help us to
generate the desired increase in the employment share of skilled labor.

The strategy of this paper is as follows: First, I will build and calibrate a
heterogenous agents DSGE model with incomplete markets and job and wage
polarization ingredients, that matches different aspects of U.S. wage and em-
ployment data in the base year of the study, which is 1981. Later, keeping the
calibration same as in 1981, I will change the price of computer capital for 2011,
which will be computed by the decline in price of technology in data over the
sample period. Comparing these two periods will enable us to make inferences
on how well the routinization hypothesis alone does in generating job and wage
polarization as observed in data. Moreover, comparing the wealth distribution
for these two years, we can analyze the contribution of job and wage polarization
processes in explaining the recently discussed middle class squeeze in terms of
wealth holdings.

The paper is organized as follows: Section 2 will provide a brief literature
review, special focus being on the papers linking job polarization and wage
inequality. Section 3 will present data and facts, while the model will be in-
roduced in Section 4. Section 5 will present our quantitative exercise, the
parameterization, solution algorithm and main findings. I will briefly discuss
the future direction of research in Section 6. Finally, section 7 will conclude.
Job polarization, which is defined as the rise in the shares of high and low skill occupations, relative to the share of middle skill occupations, has been documented for U.S. by many studies (Autor, Levy, and Murnane 2003; Acemoglu and Autor 2011; Acemoglu and Autor 2012; Jaimovich and Siu 2012; Autor and Dorn 2013). Similar employment shifts have been found for U.K. as well (Goos and Manning 2007; Bisello 2013). Furthermore, job polarization has been observed for Germany (Spitz-Oener 2006; Dustmann, Ludsteck, and Schönberg 2009; Kampelmann and Ryckx 2011) and Japan (Ikenaga and Kambayashi 2010).

The findings about the rest of the European countries are more heterogeneous. (Goos, Manning, and Salomons 2011) investigate 16 Western European countries and find a similar employment polarization between 1993 and 2006 period for most of these countries. On the other hand, (Nellas and Olivieri 2012), introducing labor market frictions in terms of unionization, find that in continental Europe, differently from the U.S. and the U.K., the fall in the share of middling paid occupations has not come with an increase in the share of low-paid employment. Similarly, (Fernández-Macías 2012) argues that rather than a pervasive process of polarization, there was a plurality of patterns of structural employment change across Europe for the period between 1995 and 2007.

As far as wage polarization is concerned, it seems more like a phenomena almost unique to U.S. For instance, (Dustmann, Ludsteck, and Schönberg 2009) and (Antonczyk, DeLeire, and Fitzenberger 2010) demonstrate that relative rise in the wages of low skill occupations observed in U.S. was not evident for Germany even though the wage movements at the top of wage distribution have been similar for 1980s and 1990s. Similarly, (Stewart 2012) and (Holmes and Mayhew 2010) show that wage of low skill occupations relative to middle skill occupations remained stable, while relative wages of high skill occupations relative to middle skill occupations grew persistently. Finally, (Massari, Naticchioni, and Ragusa 2013) conclude that there are no wage polarization trends in Europe, neither at the industry nor at the individual level.

Earlier researchers successfully explained wage inequality and increasing demand and supply of skilled labor observed in 1980s using skill biased technological change (SBTC) hypothesis. Some influential papers in this area are (Katz and Murphy 1992),(Greenwood, Hercowitz, and Krusell 1997) and (Krusell, Ohanian, Rios-Rull, and Violante 2000). However, SBTC hypothesis alone is
incapable of explaining the growth of employment share and relative wage of low skilled occupations, which has been mainly the issue of 1990s. The limitations of SBTC - an its canonical model- are further explored by (Acemoglu and Autor 2011). As a reaction, (Autor, Levy, and Murnane 2003) introduced routinization hypothesis, which is a nuanced and task-based version of SBTC to explain the phenomenon of job polarization, focusing on the impact of computerization on the different categories of workplace tasks (Bisello 2013). In a recent paper, (Autor and Dorn 2013) employ a model of changing task specialization in which “routine” clerical and production tasks are displaced by automation. The main focus of their paper is the sources of the changing shape of the lower-tail of the U.S. wage and employment distributions. They find that the twisting of the lower tail is substantially accounted for by a single proximate cause - rising employment and wages in low-education, in-person service occupations. Furthermore, they conclude that in labor markets that were initially specialized in routine-intensive occupations, employment and wages polarized after 1980, with growing employment and earnings in both high-skill occupations and low-skill service jobs.

As briefly mentioned in the introduction, routinization is not the only explanation for job and wage polarization. Even though they are less popular in the literature, other explanations include trade, globalization and offshoring, which is the relocation of some parts of production process to low wage countries. (Goos, Manning, and Salomons 2011) explore the role of technology, offshoring and institutions and conclude that technology plays the most important role in employment polarization. Other studies focusing on these alternative explanations are (Blinder 2007), (Grossman and Rossi-Hansberg 2008), (Das 2012), (Costinot and Vogel 2010), (Jung and Mercenier 2010) and (Monte 2011). Also, (Acemoglu and Autor 2011) propose a Ricardian task-based model as an alternative to the canonical model of SBTC. Recently, (Jaimovich and Siu 2012) used a search model that links job polarization to recent jobless recoveries phenomena. Their paper is among one of the few that endogeneize the skill accumulation, thereby successfully generating increase in both the supply and demand for skilled labor.
3 DATA AND FACTS

3.1 Data

The main source of wage and employment data is March Supplement of Current Population Survey (CPS) for the years between 1981 and 2011\(^2\). I used only the data for the head of household between the ages of 16 and 64. As it is common in the literature, only full time-full employment labor data has been employed. In other words, observations with working time less than 40 weeks a year and 35 hours per week were dropped. Moreover, self employed households and family workers are excluded. In addition, observations with either no income or zero income reported were eliminated. These, on average, left us around 31,000 observations per year.

The main variable of interest is the real hourly wages. To obtain this, I first generated nominal hourly wages, which is calculated as annual wage income and salary divided by total hours worked in a year. Here, total hours worked is derived by multiplying the weeks worked by the usual hours worked per week. Finally, real hourly wages are obtained by deflating the nominal hourly wages by Personal Consumption Expenditures (PCE) Index of Bureau of Labor Statistics (BLS).

CPS income data is topcoded, meaning that observations above a certain threshold are censored to ensure confidentiality. However, topcoding practice has drastically changed starting from the 1996 CPS. Prior to 1996, all the topcoded observations were being replaced by the same value that equaled to the threshold value. On the other hand, beginning from 1996, one can observe values larger than the topcode amount, since observations above the topcode amount have started to be replaced by the mean income of the families with similar characteristics. Due to this shift in topcoding practice, income data prior to and after 1996 were not comparable. To overcome this difficulty and ensure consistency between the compared periods, I kept the topcoding practice same as in pre-1996 period and replaced all observations above the top code amount with the threshold of the relevant year. It should be emphasized here that this adjustment biased the wage inequality, when measured by Gini coefficient, downward. It also surpassed the relative wages of the high skill labor compared to other skill groups. Nonetheless, the wage inequality trends still

\(^2\)Selected CPS data is reported by IPUMS. For 1981 to 2011 data, we used CPS from 1982 to 2012.
remains consistent with the existing evidence from the literature.

In addition to the adjustment at the top of the wage distribution, I also replaced the bottom 1% of the wages with 1st percentile value, with the aim of eliminating outliers and misreportings in data. Finally, I used CPS provided weights for individuals in all calculations.

3.2 Occupational Classification

I will follow (Acemoglu and Autor 2011) and (Autor and Dorn 2013) -among many other studies in the literature- to classify the occupations into different tasks. Occupations are divided along two dimensions: cognitive vs manual and routine vs non-routine. Table 1 provides detailed explanations and examples for each of these categories. In short, cognitive and manual jobs are characterized by the differences in the extent of mental versus physical activity (Jaimovich and Sin 2012). On the other hand, routine and non-routine jobs are differentiated based on the work of (Autor, Levy, and Murnane 2003) and (Autor and Dorn 2013). Routine tasks are the ones that can be summarized as a set of specific activities accomplished by following well-defined instructions and procedures. In contrast, when a task requires instant decision making, flexibility, taking initiatives, problem solving, creativity and personal interaction, then it is classified as non-routine.

Table 1. Occupational Classification

<table>
<thead>
<tr>
<th>Category</th>
<th>Non-Routine</th>
<th>Routine</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cognitive</td>
<td>Managerial, professional, technical workers (i.e. physicians, public relations managers, financial analysts, computer programmers, economists)</td>
<td>Sales, and office and administrative support, (i.e. secretaries, bank tellers, retail salespeople, travel agents, mail clerks, data entry keyers)</td>
</tr>
<tr>
<td>Manual</td>
<td>Service jobs (i.e. janitors, gardeners, manicurists, bartenders, home health aides)</td>
<td>Blue collar jobs (i.e. machine operators and tenders, mechanics, dress makers, fabricators and assemblers, cement masons, and meat processing workers)</td>
</tr>
</tbody>
</table>

For the purpose of this study- and as it is common in the literature- I classify all the routine occupations in one single category: "Routine", whether they are cognitive or manual. This group represents the tasks implemented by the "middle skill" labor. These occupations are not necessarily mechanical nor
computerized and they might even require personal interaction. The common characteristic of the tasks labeled as routine is that they can be replaced by technological improvement or can be offshored. For instance, as online sales became more widespread, many sale jobs have been lost. As another example, most of the call centers started employing labor that is physically in India.

Non-routine occupations are divided into two categories. I label cognitive non-routine occupations as "Abstract", which tends to be "high skill" labor. These are mainly managerial, professional, executive and technical jobs. Finally, manual non-routine occupations are classified as "Manual" occupations. These occupations represent the lowest tail of the skill distribution and they are termed as "service occupations" by (Autor and Dorn 2013) and (Firpo, Fortin, and Lemieux 2011). In summary, we have divided the occupations into three different tasks: Abstract, Routine and Manual, representing high, middle and low skill tasks respectively.

3.3 Facts

3.3.1 Wage inequality

Our data confirms the already documented trend in wage inequality: it has been on the rise in the past three decades. This can be seen using various measures of inequality. Figure 1 demonstrates two of them: Gini coefficient on the left and 90/10 ratio on the right.\(^3\)\(^4\) It is clear from the figure that the rise in wage inequality has been relatively modest in last decade.

\[\text{Figure 1. Wage Inequality in US}\]

\(^3\)90/10 ratio is commonly used in the literature. Unlike Gini, it is less sensitive to extremely low and high values. It is measured as the 90th percentile of income distribution divided by 10th percentile. A similar ratio, 90/50 ratio, which has special focus on the upper tail of the income distribution also reveals similar findings.

\(^4\)M_Avg represents ‘moving average’. It is the five-years moving average of the relevant variable and included to smooth the effects of survey data related problems out.
3.3.2 Job polarization

When we use the occupational classification described earlier, we see that employment has shifted from routine occupations to both abstract and manual occupations (Figure 2.a.) in last three decades. Figure 2.a., which shows the average annual percentage change in employment shares of each occupation type for different periods, reveals that job polarization trend has continued in all three decades since 1980s. In absolute terms, the share of routine occupations has declined by about 22% during the entire sample period. Meanwhile, the share of abstract jobs has increased by around 29% and the share of manual occupations has risen by almost 41%.

At this point, one can question how robust this finding is to the classification chosen. As a reaction to this, I obtained smoothed changes in employment shares by 1981 occupational skill percentile rank, using a locally weighted smoothed regression (Figure 2.b.). Similar to the case with only three groups of occupations, we see that employment shares increased in both tails, while decreasing in the middle. Hence, the existence of job polarization during the entire sample period does not seem to be contingent on a certain choice of classification.

5Employment shares are measured using the total hours worked in each group. As usual, sample weights are used.

6Wage distribution of 1981 was used as a proxy to the skill distribution. There is a strong assumption underlying this: jobs are paid according to the skill level they require. Therefore, lowest paid job as of 1981 has been assigned as the lowest skill job and so on. Sorting occupations based on their log mean wages as of 1981, employment weighted percentiles are constructed. Then, the employment shares of each bin are recalculated for 2011. Finally, using a locally weighted smoothed regression (bandwith 0.8 with 100 observations), smoothed changes in employment shares by skill percentile rank are obtained.
3.3.3 Wage Polarization

Job polarization process during the 1981-2011 period has also been coupled with polarization of the wage structure. As Figure 3 reveals, relative wages of the middle skill occupations have declined compared both to the highest and the lowest skill occupations.

**Figure 3. Wage Polarization in US: 1981-2011**

Figure 3 implies that growth in relative wages of low skill occupations was larger than that of high skill occupations. Yet, one should keep the topcoding problem in mind before reaching a firm conclusion on this. As a matter of fact, when our three-tasks classification is concerned, we see that relative mean wage of high skill occupations compared to the middle skill occupations have increased by around 16% (from 1.31 to 1.53), while the mean wage of low skill occupations relative to those of middle skill ones rose only by 6.7% (from 0.74 to 0.79) during the entire sample period.

3.3.4 College Premium and Supply of Skill

College premium, defined as the mean wage of labor with a college degree or above divided by the mean wage of labor without a college degree, has been rising during most of the sample period (Figure 4.a). This rise has been more pronounced during the 1980s and 1990s. This finding is not surprising since job and wage polarization would imply a substantial rise in the demand for skilled labor, coupled with an increase in the relative wage of this group. Meanwhile, relative supply of skilled labor - labor with a college degree and above- has increased by around 65% (Figure 4.b). The growth in the share of college educated labor has accelerated in last decade. Brought together, these two figures suggest that the increase in demand for skilled labor has been significantly larger than
the rise in the supply of skilled labor especially during the first two decades of our sample.

Figure 4. College Premium and Share of Skilled Labor

4 MODEL

4.1 Environment

Our model brings various elements of standard models of skill biased technical change (SBTC) and job polarization literatures together, taking the model of (Aiyagari 1994) as the basic starting point. Agents are heterogeneous in the sense that they receive idiosyncratic labor productivity shocks. Markets are assumed to be incomplete, meaning that agents face a borrowing constraint and thus, engage in precautionary savings to smooth their consumption.

Labor can be either of two skill groups: skilled (college) and unskilled (non-college), which are denoted as S and U respectively. There is one single final good, which is produced using both the skilled and unskilled labor in one of three tasks: Abstract \((L_a)\), Routine \((L_r)\) and Manual \((L_m)\). Along with two types of capital -computer capital \((K_c)\) and structural capital \((K_s)\)- there are five factors of production in the economy: \(L_a, L_r, L_m, K_c\) and \(K_s\). It should be noted at this point that SBTC literature misses the task based approach. On the contrary, job polarization literature abstracts from structural capital to keep the models simple and analytically tractable. Our model combines all of these in the same production function.
4.2 Households

There exists a continuum of agents of total mass equal to 1. Every period each household receives either a high (good) or low (bad) labor productivity shock. These shocks are denoted as $z_h$ and $z_l$, respectively and hence, the set of possible shocks is summarized by $Z$ such that $Z = \{z_l, z_h\}$. The productivity shocks follow a Markov process with a transition matrix of the form: $P = \begin{bmatrix} q_{ll} & 1 - q_{ll} \\ 1 - q_{hh} & q_{hh} \end{bmatrix}$, where $q_{ll}$ is the probability of staying in low state, given that we were already in low state, and $q_{hh}$ is the same for high state. In another representation: $\Pr(\ z'_t = z_l | z = z_l) = q_{ll}$ and $\Pr(\ z'_t = z_h | z = z_h) = q_{hh}$, where a variable with a prime denotes the variable for the next period. The probability of switching from one state to another is totally independent from the past history of the productivity shocks.

Each agent maximizes its preferences defined over a stochastic process for consumption given by the following utility function:

$$E[\sum_{t=0}^{\infty} \beta^t u(c_t)] , \text{ where } \beta \in (0, 1)$$

and, $u(c_t) = \frac{c_t^{1-\lambda} - 1}{1-\lambda}$

where, $\lambda > 1$ is the relative risk aversion coefficient.

Agents smooth their consumption by accumulating a single risk free asset during the good times and dissaving their assets during the bad times. We will interpret this risk free asset as a one-period-ahead sure claim on consumption goods. $a$ units of assets saved this period yields $(1+r)a$ units of consumption goods next period. In terms of general equilibrium, this asset holdings should be balanced by the structural capital demand of the producers and thus $r$ should be set in a way to equate asset demand by producers to the asset supply by households. The effective incompleteness of our model arises from the existence of credit limits. We impose the restriction that credit balances of an individual should always remain above a certain credit limit $a < 0$ and hence agents have precautionary saving motives.\footnote{As (Aiyagari 1994) argues, a borrowing limit is required as long as $r < 0$. Otherwise the problem will not be well posed and there will not exist a solution. Since the present value of life time earnings is infinite, nothing will prevent the agents from running a Ponzi scheme. On the other hand, when $r > 0$, a less restrictive borrowing constraint would suffice. For instance, imposing present value budget balance, along with nonnegativity of consumption necessarily imply a borrowing constraint.}

Each worker $i$ is endowed with a vector of three efficiency sets: $E_i = (a_i, r_i, m_i)$, where $a_i$ represents the efficiency of the agent in abstract task and
so on. We are assuming following efficiency vectors for skilled and unskilled labor respectively: \( E_{S,i} = (\xi, 0, 0) \) and \( E_{U,i} = (0, \eta_i, 1) \) where subscripts \( S \) and \( U \) indicate whether the variable in question belongs to skilled or unskilled labor. These vectors imply that only skilled labor can work in abstract tasks and also skilled labor cannot work in routine nor manual tasks. Since efficiency of a skilled labor in abstract tasks, \( \xi \), is the same across all agents in this group, they all receive the same wage rate: \( w_a \).

Unskilled labor can supply one unit of labor either to routine or to manual tasks. Once employed in manual task, all the agents are homogenous, meaning that they have the same efficiency and receive the same wage: \( w_m \). On the other hand, when they are employed in routine task, they are heterogenous with agent-specific relative efficiency: \( \eta_i \). Wages in routine tasks are also different for each agents since they will receive \( w_r \eta_i \). Routine task efficiency units are assumed to be distributed with density \( f(\eta) \), such that \( \int_0^\infty f(\eta) d\eta = 1 \).

Labor supply decision of skilled labor is quite standard as she inelastically supplies one unit of labor to abstract task and receives \( w_a \). On the other hand, unskilled labor supplies routine labor if her wage in routine task exceeds that in manual task: \( w_r \eta_i \geq w_m \). This decision rule yields a cutoff efficiency level, \( \eta^p = \frac{w_m}{w_r} \), above which unskilled agent supplies all of its labor to routine task.

Within this framework, one period budget constraint of the skilled and unskilled agent becomes:

\[
S: \quad c_S + a_S' \leq (1 + r)a_S + w_a l_a z
\]

\[
U: \quad c_U + a_U' \leq (1 + r)a_U + \max\{w_r \eta_i l_r, w_m l_m\} z
\]

\( c \geq 0 \) and \( a \geq a_0 \).

Here, \( a' \) stands for \( a \) in next period.

To incorporate a simple skill choice to our model, I follow the approach from (Blanchard 1985) and (Zambrano 2013) introduce death probability. Every period, households die with a probability of \( \psi \). The dead household is immediately replaced by a newcomer, who inherits the assets of the dead. The newcomer draws a routine task efficiency endowment \( \eta_i \) from the probability distribution \( f(\eta) \). The newcomer is assumed to have the same productivity level as the dead and follows the same Markov process. Observing her efficiency, inherited asset level and the fixed cost of education (\( \varphi \)), the newcomer makes a choice whether
to acquire a college degree or not if she has enough resources for that. In other words, if the newcomer does not have enough resources to cover education expenses, she automatically becomes unskilled. Therefore, the value function at the beginning of the life of a newcomer $i$ will be:

$$v_0(a', z'; h, e, r, w) = \max \{ v(a', z'; U, e, r, w), v(a' - \varphi, z'; S, e, r, w) \}$$

if $a - \varphi \geq -a$, and

$$v_0(a', z'; h, e, r, w) = v(a', z'; L, e, r, w)$$

otherwise.

where $e$ represents the efficiency endowment vector, $r$ is the interest rate, $h$ represent the skill level such that $h \in \{S, U\}$ and $w$ is the set of wages $\{w_a, w_r, w_m\}$.

In summary, altruistic agents that maximize lifetime utility of the household will have the following value function:

$$v(a, z; h, e, r, w) = \max_{c, a'} \{ u(c) + \beta \psi[P(z, z)v(a', z; h, e, r, w) + (1 - P(z, z))v(a', z'; h, e, r, w)] + \beta(1 - \psi)[P(z, z)v_0(a', z; h, e, r, w) + (1 - P(z, z))v_0(a', z'; h, e, r, w)] \}$$

subject to

$$c + a' \leq (1 + r)a + w(h, e)l_{h,e}z$$

$$c \geq 0 \text{ and } a' \geq g$$

Zambrano (2013) provides a lemma (Lemma 2) that suggests that if the cost of education is lower than a treshold, only poor individuals that can afford it will attend college. Similarly, if the cost of college is larger than an upper treshold, only the rich individuals will attend college. When the cost of education is between these two tresholds, then individuals with asset holdings that are not neither too high nor too low will attend college. This lemma follows from the concavity of value function as well as the fact that $v(a', z'; S, r, e) - v(a', z'; U, r, e)$ is decreasing in assets.
From now on, I will assume that all the assumptions and theorems in (Stokey and Lucas 1989) that are necessary for this process to have a unique solution are valid. Let the state vector describing an agent’s position at a time be denoted as $X = (a, z)$ such that $x \in X$ where $A = [a, \infty]$ and $Z$ is as described above. Thus, assuming that a bounded measurable solution $v$ to the equation (1) exists, then that $v$ is the optimal value function. Then, the functions $c : X \times R_{++} \rightarrow R_{+}$ and $a : X \times R_{++} \rightarrow A$ are optimal decision rules provided that $c(x; h, e, r, w)$ and $a(x; h, e, r, w)$ are measurable, feasible, and satisfy the value function $v(x; h, e, r, w)$.

4.3 Final Good Producing Firms

Final good is produced by employing five factors of production mentioned earlier by perfectly competitive firms renting skilled and unskilled labor and structural and computer capital. I incorporate the task-based approach of the job polarization literature into an otherwise standard production function that is common in the SBTC literature - to be more specific, the production function of (Krusell, Ohanian, Rios-Rull, and Violante 2000):

$$Y = K^{\alpha} a \{ \mu L_r^\rho + (1 - \mu) [\omega (\xi L_a)^\rho + (1 - \omega) K_c^\rho]^{\sigma / \rho} \}^{\gamma / \rho} L_m^{1 - \gamma - \alpha} \quad (2)$$

where, $\alpha, \gamma \in (0, 1)$ and $\alpha + \gamma < 1$ are shares of structural capital and non-manual tasks combined with computer capital, respectively. Also $\mu$ and $\omega$ are the parameters that govern the income shares, while $\rho$ and $\sigma$ govern the elasticity of substitution across factors.

This production function yields the following cross-elasticities between factors:

$$\varepsilon_{L_a, K_c} = \frac{1}{1 - \rho}$$
$$\varepsilon_{L_r, K_c} = \frac{1}{1 - \sigma}$$
$$\varepsilon_{L_m, K_c} = 1$$

If the routinization hypothesis is valid, $K_c$ and $L_a$ should be relative complements ($\varepsilon_{L_a, K_c} < 1$), while $K_c$ and $L_r$ are relative substitutes ($\varepsilon_{L_r, K_c} > 1$). Furthermore, $K_c$ will be less complementary with $L_m$ than with $L_a$ and less
substitutable with $L_m$ than with $L_r$ since $\varepsilon_{L_m,K_c} = 1$. The fact that $K_c$ will be less complementary with $L_m$ than with $L_a$ is a more realistic assumption than most of the studies in job polarization literature, which usually assumes that computer capital complements both abstract and manual labor in an equal degree.

When elasticities satisfy the restrictions above, a decrease in the price of computer capital will reduce demand for routine task, while increasing the demand for other two. This, in turn, will put downward pressure on the relative wages paid to routine task compared to other two tasks.

### 4.4 Computer Capital Producers

As it is standard in job polarization literature, computer capital ($K_c$) is produced by perfectly competitive firms, using some part of the final good. Production technology is:

$$K_c = \frac{Y_c e^{\nu_t}}{\theta}$$

where, $Y_c$ is part of final good used in production of computer capital. Hence, we have $Y = Y_c + C + I_s$, where $I_s$ is the investment on structural capital. Furthermore, $\nu_t$ is the growth rate of computer capital productivity (or equivalently rate of decline of computer capital price), while $\theta = e^{\nu_0}$ is an efficiency parameter. We also assume that computer capital fully depreciates between periods ($\delta_c = 1$). This assumption is not as unrealistic as it seems at first glance when computer capital is interpreted as computer and information technology a firm rents each year. Thus, price paid to this good can be interpreted as rent paid to technology a firm hires.

Perfect competition guarantees that:

$$p_c = \frac{Y_c}{K_c} = \theta e^{-\nu_0}$$

Since we assumed that $\theta = e^{\nu_0}$, initial price of computer capital will equal to 1 ($p_c = 1$). In this paper, we will not make $\nu$ time dependent. Rather, we will introduce technological change by an exogenous increase in $\nu$, or an exogenous fall in $p_c$. This, as explained earlier, will trigger the polarization process as cheaper computer replaces routine tasks and induce a higher demand for other tasks.
4.5 Structural Capital

Structural capital \((K_s)\) in the model is the standard capital in most of the economic models. It can be produced from final output on a one-to-one basis. The stock of structural capital evolves according to standard law of motion:

\[
K'_s = (1 - \delta_s)K_s + I_s
\]

where \(0 < \delta_s < 1\) is the depreciation rate.

Structural capital is assumed to be owned by households and rented to the final good producers at the rate \(r + \delta_s\).

4.6 Summary of the Variables

Table 2 summarizes the basic variables of interest in our model. We will keep the price of computer capital as exogenous and investigate the effects of changing it on wages and employment shares.

<table>
<thead>
<tr>
<th>Endogenous</th>
<th>Exogenous</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wages</td>
<td>Price of computer capital</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>Efficiency endowments</td>
</tr>
<tr>
<td>Factor demands</td>
<td></td>
</tr>
<tr>
<td>Cutoff efficiency</td>
<td></td>
</tr>
<tr>
<td>Supply of skilled labor</td>
<td></td>
</tr>
</tbody>
</table>

5 QUANTITATIVE EXERCISE

In this section, I will solve the model quantitatively for two different states and test how well routinization hypothesis does in explaining the employment and wage shifts observed in data. To begin with, I will parameterize the model targeting task shares, wage ratios and factor income shares for the year 1981. Then, price of computer capital will be estimated for year 2011, while all the parameters are kept the same. However, both the borrowing limit and cost of education will be adjusted for 2011 to keep the results comparable. Finally, supply of skills, employment shares and asset distribution will be compared for two periods.
5.1 Parameterization

5.1.1 Household

Parameters related to the household problem will mainly be borrowed from (Huggett 1993) and (Aiyagari 1994). Unlike (Huggett 1993), who follows the approach of (Imrohoroglu 1989) and interprets $z_h$ and $z_l$ as earnings when employed and not employed, we will follow (Aiyagari 1994) and assume that idiosyncratic shocks are productivity shocks. Thus, we will set $z_h = 1$ and $z_l = 0.4$ following (Aiyagari 1994). Consistent with this, we will use the transition matrix implied by (Aiyagari 1994): $P = \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{bmatrix}$, which yields invariant probability distribution $\pi = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$, meaning that agents have high productivity half of the time. Note that one period in our model is 1 year.

As it is standard in the literature, discount factor $\beta$ is set as 0.96. Following (Mehra and Prescott 1985), the coefficient of relative risk aversion $\lambda$ is set to 1.5. Regarding the credit limit, we set it equivalent to the average income of one year. To keep the results comparable between two years, we adjust $z_2$ to be equal to one year’s average income when we change the price of computer capital. Death rate is set as 0.0083 from data as 0.83% of the total population die every year in U.S.

A key parameter to set is $\varphi$, cost of education. In this exercise I set it equal to 3.16 times the median wage. This follows from the 2012 College Board Report estimate for the cost of four-years college in a private nonprofit university. When this parameter changes, the share of agents who obtain education is affected significantly. Therefore, results are not robust to choice of $\varphi$ in terms of absolute values obtained. For instance, if we had used cost of public universities instead (1.86 instead of 3.16), a larger share of agents would prefer getting education and this would affect employment shares and relative wages. However, even in that case, all the key variables would still change in the same direction following a decline in the price of computer.

Following the job polarization literature (Autor and Dorn 2013), we assume that routine task efficiency endowments have an exponential distribution with parameter $\kappa$: $f(\eta_i) = \kappa e^{-\kappa \eta_i}$ for $\eta_i \in [0, \infty)$. This distribution (with $\kappa = 1$) is used extensively in the literature since exponential distribution is easy to solve analytically. For our purpose, we do not need exponential distribution and it
should be disciplined by data since choice of distribution might affect some of the findings. Yet, when Pareto distribution has been used instead of exponential distribution, main messages of the paper remains the same.

Table 3 summarizes the parameters of the household problem. At this point, κ is missing and it will be obtained in the following subsections.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor β</td>
<td>0.96</td>
<td>Huggett (1993)</td>
</tr>
<tr>
<td>Death rate ψ</td>
<td>0.0083</td>
<td>Data</td>
</tr>
<tr>
<td>CRRA coefficient λ</td>
<td>1.5</td>
<td>Mehra and Prescott (1985)</td>
</tr>
<tr>
<td>Borrowing limit γ</td>
<td>w_average</td>
<td>Model</td>
</tr>
<tr>
<td>Cost of Education φ</td>
<td>3.16 × w_median</td>
<td>Data</td>
</tr>
<tr>
<td>Transition Matrix P</td>
<td>[0.8, 0.2]</td>
<td>Aiyagari (1994)</td>
</tr>
<tr>
<td>Productivity Endow. ε</td>
<td>[1, 0.4]</td>
<td>Aiyagari (1994)</td>
</tr>
</tbody>
</table>

5.1.2 Firms

Most of the parameters of the production side are taken from the SBTC literature. Share of structural capital is estimated by (Krusell, Ohanian, Rios-Rull, and Violante 2000) as 0.117. They also estimate elasticity of substitution between equipment capital and unskilled labor as 1.6694 and between equipment capital and skilled labor as 0.6689. Even though they do not have task based approach, I will assume here that their skilled labor and our abstract task, which could be implemented only by skilled labor, can be used interchangeably. Following this assumption, we can assume that their unskilled labor and our routine task match to a great extent as well.

It should be noted here that the equipment capital in (Krusell, Ohanian, Rios-Rull, and Violante 2000) is not exactly the same as our computer capital here since we assume it totally depreciates every period, but their equipment capital accumulates even though it appreciates much faster than the structural capital. Nevertheless, the equipment capital in (Krusell, Ohanian, Rios-Rull, and Violante 2000) and (Greenwood, Hercowitz, and Krusell 1997) covers computer capital as well. Therefore, I will assume that two definitions can be interpreted as the same variable. Thus, elasticity of substitution between routine task and computer capital is set to 1.6694 and that between computer capital and abstract task is set to 0.6689. Note that these parameters make abstract
task and computer capital relative complements, while routine task and computer capital relative substitutes as routinization hypothesis requires.

Following (Greenwood, Hercowitz, and Krusell 1997), I choose 0.05 as the depreciation rate of structural capital. As mentioned earlier, $\delta_c$ is assumed to be equal to $1.9$

We had assumed that price of computer capital is assumed to be equal to 1, which is also the price of final good. For 2011, we need to calculate the price of computer capital again since we exogenously change it to compare two periods. To obtain $p_c$ for 2011, we need to find the growth rate of productivity of computer capital or deflation rate of computer capital price. Starting from 1988, I used Information Technology, Hardware and Services Index of BLS, which decreased 9.7% on average between the years 1988 and 2011. Prior to 1988, I used estimate of (Greenwood, Hercowitz, and Krusell 1997) for the growth of equipment capital productivity, which is 3.24%. At first glance, these two values might seem incomparable. However, the decline rate of Information Technology, Hardware and Services Index in the early years is close to 3.24% as well. The decline rate in index jumps to very high levels in 90s and 2000s, thereby increasing the average to 9.7%.

Table 4 summarizes the parameters chosen for the problem of firms.

<table>
<thead>
<tr>
<th>Table 4. Parameters of the Producer Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$ Share of $K_S$ $0.117$ Krusell et al. (2000)</td>
</tr>
<tr>
<td>$\delta_c$ Depreciation rate $0.05$ Greenwood et al. (1997)</td>
</tr>
<tr>
<td>$\frac{1}{1-\varepsilon}$ $\varepsilon$ b/w R &amp; $K_c$ $1.6694$ Krusell et al. (2000)</td>
</tr>
<tr>
<td>$\frac{1}{1-\rho}$ $\varepsilon$ b/w A &amp; $K_c$ $0.6689$ Krusell et al. (2000)</td>
</tr>
<tr>
<td>$\nu$ % Decline in $p_c$ $3.24%$ until 1988 Greenwood et al. (1997)</td>
</tr>
<tr>
<td>$9.7%$ from 1988 BLS</td>
</tr>
</tbody>
</table>

5.1.3 Missing and Estimated Parameters

After the parameterization described so far, we are still missing five parameters: $\mu, \omega, \gamma, \xi, \kappa$. The first two of them are the parameters governing the income shares. $\gamma$ is the share of three factors of production combined ($L_a$, $L_r$ and $K_c$) in the production function. $\xi$ is the efficiency of abstract labor in production. Finally, $\kappa$ is the parameter of the exponential distribution that governs the

---

9(Greenwood, Hercowitz, and Krusell 1997) estimate the depreciation rate of equipment capital as 0.125.
routine task efficiency distribution. To estimate these parameters, we have six
targets to match from the data. Those targets are summarized as follows:

\[
\begin{align*}
\text{Share}\_\text{Manual} &= 0.13 \quad (3) \\
\text{Share}\_\text{Abstract} &= 0.29 \quad (4) \\
\frac{w_A}{w_R} &= 1.31 \quad (5) \\
\text{Capital}\_\text{Income}\_\text{Share} &= 0.3 \quad (6) \\
\eta^* &= \frac{w_M}{w_R} = 0.71 \quad (7) \\
\xi &= E(\eta) \times 0.9333^{**} \quad (8)
\end{align*}
\]

Target (8) follows from the estimate of (Acemoglu 2002) for the year 1980. Until mid 1980s, skilled labor was less efficient than unskilled labor. Over time, both the relative and absolute efficiency of unskilled labor fell considerably. Estimate of (Acemoglu 2002) is also consistent with that of (Caselli and Coleman 2006) and (Unal 2010).

Note that we have six targets to match but five parameters to estimate. Therefore, we have to release one of the targets. Fortunately, when we drop target (4), we estimate parameters yielding a very close value for the target (4) as well. It should be noted here that I assumed that 33.2\% of the agents are skilled as it is observed in CPS data for the year 1981 when estimating the missing parameters.\(^{11}\) Later, when solving the model, I will let the share of skilled labor to change endogenously and see if we can obtain a share close to 33.2\% or not.

Table 5 presents the results of our estimation.

<table>
<thead>
<tr>
<th>Table 5. Estimated Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
</tr>
<tr>
<td>$\omega$</td>
</tr>
<tr>
<td>$\kappa$</td>
</tr>
<tr>
<td>$\gamma$</td>
</tr>
<tr>
<td>$\xi$</td>
</tr>
</tbody>
</table>

\(^{10}\)Here, I implicitly assumed that skilled labor of (Acemoglu 2002) matches our abstract labor, while his unskilled labor coincides with our routine labor.

\(^{11}\)I assume that college graduates refer to "college equivalent" definition of (Autor, Krueger, and Katz 1998). College equivalent is defined as those with a college degree +0.5x those with some college.
5.2 Algorithm

In this section, algorithm used to solve the quantitative model is summarized briefly. We have two loops; outer loop being for the interest rate $r$, while inner loop is over the relative wage rate $\frac{w_a}{w_r}$. We begin with an initial guess of $r$, that covers the likely outcomes of interest rate.\footnote{According to (Aiyagari 1994), this interest rate should lie between complete markets interest rate $\left(\frac{1}{y}\right)$ and $-\delta_S$.} We also pick an initial guess for relative wage between abstract and routine tasks. The range chosen is between 1 and 3. Employing our initial guesses of $r$ and $\frac{w_a}{w_r}$, we solve for optimal decision rules for asset holdings using the Value Function Iteration (VFI) method.

Once, we have the decision rules, we simulate the economy for 1,000,000 agents to compute invariant asset distribution and share of agents willing to get college education. Using this share of skilled labor, we compute the implied wage ratio. If this ratio is larger than our initial guess, we increase our guess for $\frac{w_a}{w_r}$ using the bisection method. Similarly, when the implied wage ratio is lower, we decrease our initial guess. Once $\frac{w_a}{w_r}$ is updated, we repeat the VFI and simulation parts again until $\frac{w_a}{w_r}$ converges.

After we obtain convergence in $\frac{w_a}{w_r}$, we check for the excess demand for asset holdings. Demand for holdings comes from the firm’s choice of structural capital. On the other hand, supply of asset is obtained from the simulation. If excess demand is positive, we update $r$ upward using bisection method. Likewise, negative excess demand requires updating $r$ downward. We repeat the whole process until $r$ converges (or until excess demand for asset holdings is around 0).

5.3 Findings

5.3.1 Job and Wage Polarization

Our model does a remarkably good job in matching the data for the year 1981 (Table 6). This results from the ability of the model to generate share of skilled labor matching the college equivalent in data we used in our parameter estimation. Once the share of skilled labor has been predicted correctly, rest of the variables match the data since the parameters have already been estimated for that particular level of skilled labor share. Model is also fairly successful in predicting the rise in the share of abstract jobs, even though it overshoots it a little. Rather than increasing to 39% as in data, share of abstract jobs increase
to 44.4% in the model. On the other hand, the direction of wage polarization in the model is the same as in data. However, relative increase in the wage of abstract tasks is significantly above that in data. This mainly stems from the fact that model falls short of data in terms of the share of college equivalent workers. This results in higher wages for the abstract task since supply of skilled labor is not as abundant as in data.

Table 6. Findings: Data vs Model

<table>
<thead>
<tr>
<th></th>
<th>1981</th>
<th></th>
<th>2011</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>Share_L_A</td>
<td>29</td>
<td>28.8</td>
<td>39</td>
<td>44.4</td>
</tr>
<tr>
<td>Share_L_R</td>
<td>58</td>
<td>58.1</td>
<td>44</td>
<td>44.0</td>
</tr>
<tr>
<td>Share_L_M</td>
<td>13</td>
<td>13</td>
<td>17</td>
<td>11.5</td>
</tr>
<tr>
<td>(w_A )</td>
<td>1.31</td>
<td>1.31</td>
<td>1.56</td>
<td>1.79</td>
</tr>
<tr>
<td>(w_R )</td>
<td>0.71</td>
<td>0.71</td>
<td>0.81</td>
<td>0.84</td>
</tr>
<tr>
<td>Share_College*</td>
<td>33.2%</td>
<td>33.12%</td>
<td>53.3%</td>
<td>49.6%</td>
</tr>
</tbody>
</table>

As both Table 6 and Figure 5.a. demonstrate, the main failure of the model is its inability to generate the rise in the share of manual occupations. On the contrary, share of this group even declines in the model. There might be two explanations for this failure. First, exponential distribution assumption for efficiency endowments might not be representative of the actual data. When the relative wage of the manual jobs increase in our model (due to the increased demand for manual task), this does not generate a sufficient increase in the supply of manual task. Second, our model misses an additional demand channel for the manual labor. According to the routinization hypothesis, when the demand for and supply of abstract task increase, coupled with an increase in relative wage of this group, there will be higher demand for manual jobs such as cleaners, baby-sitters, janitors and similar jobs. Our model misses this important channel. Instead, we are assuming that the demand for manual tasks directly increases due to the fall in price of computer capital, which is a relatively weaker channel. In order to overcome this difficulty, we might incorporate another sector (service sector) such as the one in (Autor and Dorn 2013) that employs only the manual task. Having two goods in utility function, with appropriate choice of elasticity of substitution between them, might generate a stronger demand
channel for the manual jobs. The effect of this increased demand for abstract jobs on the relative wages of this group can be offset partially (or totally) by the likely increase in supply of this skill group, which also implies a decline in the supply of routine labor. Overall, we might obtain the desired rise in share of manual tasks, while keeping the increase in relative wage of low skill occupations compared to middle skill occupations the same.

Figure 5. Job and Wage Polarization: Data vs Model

5.3.2 Wealth Distribution
Comparing the asset distributions, we can see that our model turned out to be quite successful in capturing the ‘middle class squeeze’. As seen in Figure 6, there are more agents with negative wealth holdings after the exogenous shock to the price of computer capital and also the right tail of the wealth distribution is longer, while the erosion in the middle of wealth distribution is striking. This might be an indication of the likely role played by job and wage polarization triggered by the rapid decline in computer price on the ongoing squeezing of the middle class.

Figure 6. Invariant Asset Distribution (1981 vs 2011)
5.3.3 Robustness

As mentioned earlier, the findings presented so far are sensitive to the choice of cost of education parameter $\varphi$. When we use public education instead of private nonprofit colleges to estimate a value for $\varphi$, more agents choose getting education in both steady states. Therefore, we are not able to match the wage ratios and employment shares in the data in absolute terms. Nonetheless, the direction of the changes in the employment shares and relative wages remain the same even with a low value for $\varphi$. Thus, the ability of the model to generate wage polarization and increase in the share of abstract occupations consistent with data and its inability to capture the rise in the share of manual occupations are robust to the choice of education parameter.

Findings concerning the employment share and relative wage of manual occupations are also sensitive to the choice of skill distribution. In our model, we assumed that skill distribution is exponential with parameter $\kappa$. Even though the model could generate the increase in relative wage of manual occupations, it failed to mimic the rise in the share of this group in employment. When we use Pareto distribution instead, this time we can generate a slight increase in the employment share of manual occupations. However, in this case, model fails to capture the rise in the relative wage of this group. Therefore we face the same problems mentioned earlier: skill distribution should be disciplined better and also there should be a stronger channel linking abstract and manual occupations, thereby generating a higher demand channel between computer price and manual occupations.

6 FUTURE DIRECTION OF WORK

As it has already been discussed, the calibration of the skill distribution should be disciplined by data. This might solve the inability of the model to generate rise in employment share of manual occupations (exponential distribution) or the rise in relative wages of this group (Pareto distribution). However, the main problem here is the difficulty of measuring the labor efficiencies in a particular group of occupations (routine) from the survey data.

In our model, we assumed that price of computer capital was exogenous and it changed abruptly between two periods. In a more realistic set up, we can introduce the price of computer capital as a stochastic process, which can be interpreted as the aggregate uncertainty in the economy. This process can be
calibrated using the time series data from BLS for the technology prices. When the price of computer capital is uncertain, it might affect the saving behavior of the agents since uncertain computer price would bring uncertainty about the wages the agents will face. This will also affect their choice of investing in college education or not.

Another important extension would be to incorporate multiple sectors into our baseline model. As we already discussed it, having a service sector that uses only manual labor, might solve the problem of the model to generate increase in employment share of manual occupations, if the service is substitute with the other good that uses computer capital, structural capital and abstract and routine labor. In addition to this, a multi sector model, in which sectors are heterogenous with respect to their routine task intensities might do well in explaining the differences across sectors in terms of wage inequality patterns. Figure 7 shows that sectors that had high routine task share as of 1975, experienced much larger increases in within sector wage inequality over the sample period. Incorporating a multi sector setup and heterogeneity across sectors into our baseline model might reveal the role played by the rapid decline in price of computer capital and the accompanying job and wage polarization in explaining the sectorial differences depicted in Figure 7.

**Figure 7. Change in Within Sector Wage Inequality**

---

7 EXTENSION TO TWO-SECTORS MODEL

As already discussed, our model with only one sector failed to replicate some key aspects of the data; especially in generating the desired increase in share of manual tasks. We attributed this to the lack of a stronger and more realistic link between price of technology and demand for services. To overcome this, we incorporate a second sector into the baseline setup introduced in earlier sections. Following (Autor and Dorn 2013), we assume that this second sector only employs low skill labor, who work in manual tasks, and produces service occupations. As long as consumption elasticity between the final good and service is sufficiently small, a rise in computer price will generate the desired flow of low skill labor from routine tasks to manual tasks.

7.1 Amendments to the Model

The final good production technology is the same as before, except that we only allow use of four factors of production: \( L_a, L_r, K_c \) and \( K_s \), rather than five. In other words, we rule out the use of manual task in goods production technology. Also, we will normalize \( \xi \), the efficiency of high skill labor in abstract tasks, to 1, as we will rely on comparative advantages of each labor type in routine tasks to allocate labor in this version of the model. The final good production technology then becomes:

\[
Y_g = K_s^\alpha \left\{ \mu L_r^\sigma + (1 - \mu) [\omega L_a^\rho + (1 - \omega) K_c^\rho]^{\sigma/\rho} \right\}^{(1-\alpha)/\sigma}
\]

where \( Y_g \) is the final good produced. As in our one-sector model, abstract labor and computer capital will be relative complements, while abstract labor and routine labor will be relative substitutes with our calibration. The parameters governing the factor income shares will be re-calibrated below trying to match income shares, wage ratios and employment shares.

Final good is our numeriare good and hence its price is normalized to 1. The first order conditions of firm’s profit maximizing problem are:
Note that $L_r$ is the aggregate routine task labor in terms of efficiency units.

The service sector uses only manual labor and have a simple linear production function as follows:

$$Y_s = \alpha_s L_m$$

where $Y_s$ is the total output of service sector and $\alpha_s$ is efficiency of manual labor, which will be normalized to 1.

The profit maximization of the service sector firm yields a simple condition:

$$p_s = w_m.$$ 

Computer capital production technology is same as before and price of computer capital is initially set equal to the price of final good.

As before, we are assuming that there is a continuum of mass one of both skilled (college educated) and unskilled labor. However, unlike (Autor and Dorn 2013) and our baseline model, we are assuming that skilled workers can work both in abstract routine tasks but not in manual tasks. On the contrary, unskilled workers are not allowed to work in abstract tasks but can do both manual or routine jobs. Thus, efficiency endowment vectors are: $E_{S,i} = (1, \omega, 0))$ and $E_{U,i} = (0, \eta_i, 1)$ where subscripts $S$ and $U$ represent skilled and unskilled respectively. In our calibration, the average efficiency endowment in routine tasks will be lower for the skilled workers than that of unskilled workers. Therefore, unskilled labor will have a comparative advantage in routine tasks, as expected and thus, most of the skilled labor in routine tasks will leave that market after a significant decline in price of technology, leaving only the labor with very high relative efficiency in routine tasks.

Allowing the skilled labor to work in routine tasks as well is not unrealistic as it seems. As a matter of fact, CPS data between 1981 to 2011 reveal that on
average 20% of college equivalent labor works in non-abstract tasks. Similarly, around 26.6% (from 16.8% to 35.3%) of the workers employed in non-abstract occupations have a college equivalent degree. This new set up will enable us to generate shifts in employment shares even in the absence of educational choice set up described in the baseline model. The main goal of this modification is to reduce the computation time substantially.

As before, skilled labor working in abstract tasks receive the same wage rate; $w_a$, since efficiency of a skilled labor in abstract tasks is the same across all agents. However, when a skilled labor works in routine task, she is paid based on her efficiency and thus receive $w_r \varpi_i$. The skilled labor faces a self selection problem: she chooses the highest paying task. To be more explicit, skilled worker chooses staying in abstract task if $w_r \varpi_i < w_a$ and will go and work in routine task otherwise. Similar to the problem of unskilled labor described earlier, this selection problem will generate a cutoff $\varpi^* = \frac{w_a}{w_r}$. Thus, an skilled labor will work in routine task only if her efficiency in routine tasks is very high. In other words, only the skilled labor with high comparative advantage in routine tasks will leave the abstract task market.

The problem of unskilled labor is same as before. Unskilled labor supplies routine labor if her wage in routine task exceeds that in manual task: $w_r \eta_i \geq w_m$. This decision rule yields a cutoff efficiency level, $\eta^* = \frac{w_m}{w_r}$, above which unskilled agent supplies all of its labor to routine task.

As before, routine task efficiency units are assumed to be distributed with density $f(\eta_i)$, such that $\int_0^\infty f(\eta_i)d\eta_i = 1$. In addition to this, we now have a similar distribution for $\varpi_i$ such that $\int_0^\infty g(\varpi_i)d\varpi_i = 1$. We will assume -and indeed our calibration will require- that $E(\eta_i) > E(\varpi_i)$, meaning that unskilled labor has comparative advantage in routine tasks over the skilled labor.

Following (Autor and Dorn 2013), consumers have the following utility function defined over two goods: services and goods:

$$u = \left(\gamma c^\lambda_a + (1 - \gamma)c^\lambda_g\right)^\frac{1}{\lambda}$$

where $\lambda$ and $\gamma < 1$.

Here, elasticity of substitution in consumption between goods and services is $\frac{1}{\lambda-1}$. This elasticity plays a crucial role in shaping the employment shifts following the decline in price of technology. On the other hand, the parameter $\gamma$ is a scaling factor that helps us to match the key facts targeted in our calibration.

One period budget constraints in this set up are as follows:
\[ S : \quad c_{s,g} + p_s c_{s,s} + a_S \leq (1 + r)a_S + \max\{w_a, w_r \sigma_i\}z \]

\[ U : \quad c_{U,g} + p_s c_{U,s} + a'_U \leq (1 + r)a_U + \max\{w_r \eta_i, w_m\}z \]

where \( c_{I,j} \geq 0 \) and \( a_I \geq 0 \) for \( I \in \{S, U\} \) representing skilled and unskilled labor and \( j \in \{s, g\} \) representing goods and services.

Letting \( C_g \) and \( C_s \) be the aggregate consumption of goods and services respectively, first order conditions of household problem yields:

\[ \left( \frac{C_g}{C_s} \right)^{1 - \lambda} = \frac{1 - \gamma}{\gamma} p_s \tag{9} \]

We also have the standard Euler equations for each type of the household.

The market clearing conditions for goods and services are as follows:

\[ Y_g = C_g + I_S + K_c \]

\[ Y_s = C_s \]

Finally, the labor market clears:

\[ L_a = \int_0^{\sigma^*} f(\sigma_i) d\sigma_i \]

\[ L_r = \int_{\eta^*}^{\infty} \sigma_i f(\sigma_i) d\sigma_i + \int_{\eta^*}^{\infty} \eta_i f(\eta_i) d\eta_i \]

\[ L_m = \int_0^{\eta^*} f(\eta_i) d\eta_i \]

### 7.2 Calibration

Our calibration strategy is same as before. For the household side, we have the transition matrix and productivity endowments as in (Aiyagari 1994) and discount factor \( \beta \) as in (Huggett 1993). Elasticities of substitution between abstract and routine task and computer capital as well as the share of structural capital are taken from (Krusell, Ohanian, Rios-Rull, and Violante 2000). Finally, we take the depreciation rate of structural capital from (Greenwood, Hercowitz, and Krusell 1997) as in the baseline model.
To calibrate the elasticity of substitution between goods and services, we use consumption and price series for goods and services from the Bureau of Economic Analysis. For services, we take only personal care and clothing services item since it best represents the manual services in our model. However, results are robust to other choices of service items such as household maintenance services. We do not use the general service category since most of the subcategories under services such as financial, education and health services coincides with our abstract tasks in the model and hence are considered in goods. To calibrate the elasticity of substitution, we used equation 9 and regressed \( \ln\left(\frac{C_g}{C_s}\right) \) on a constant and \( \ln\left(\frac{1}{p_s}\right) \) and interpreted the coefficient of the explanatory variable, which is 0.6883068, as the elasticity of substitution between goods and services.\(^4\) This implies \( \lambda = -0.452843 \).

Finally, we used the same strategy as in the baseline to calibrate the remaining parameters: \( \omega, \mu, \kappa_1 \) and \( \kappa_2 \), where the last two are the parameters of exponential distribution governing the routine task efficiency distribution for unskilled and skilled labor respectively. To calibrate these four parameters, along with the scaling parameter \( A \), we target five facts from data and solve them along with the other steady state equations:

\[
\text{Share Manual} = 0.13 \tag{10}
\]
\[
\text{Share Abstract} = 0.29 \tag{11}
\]
\[
\varpi^* = \frac{w_A}{w_R} = 1.31 \tag{12}
\]
\[
\eta^* = \frac{w_M}{w_R} = 0.71 \tag{13}
\]
\[
\text{Capital Income Share} = 0.3 \tag{14}
\]

When we solve these equations for \( p_k = 1 \), we obtain the following parameters:

\[\text{Table 7. Estimated Parameters of 2-Sectors Model}\]

\[^{14}\text{When we take log of both sides, equation 9 becomes:}\]
\[
(1 - \lambda) \ln\left(\frac{C_g}{C_s}\right) = \ln\left(\frac{\gamma}{1 - \gamma}\right) - \ln\left(\frac{1}{p_s}\right)
\]
\[
\ln\left(\frac{C_g}{C_s}\right) = \text{constant} - \frac{1}{1 - \lambda} \ln\left(\frac{1}{p_s}\right)
\]
This parameterization ensures that unskilled labor has comparative advantage in routine task over the skilled labor since $E(\eta_i) > E(\omega_i)$. This is also depicted in Figure 8, which shows the distribution of efficiency in routine task for two skill groups.

**Figure 8. Efficiency Distribution in Routine Tasks: Skilled vs Unskilled**

7.3 Preliminary Findings

In this subsection, we follow the same strategy as in the baseline model and compare two steady states that only differ in price of computer technology. Figure 9 is a replication of Figure 5, using the two-sectors model instead of our baseline one-sector model. However, the findings presented in Figure 5 are partial equilibrium results and the complete general equilibrium model is yet to be solved. Nevertheless, the change in interest rates do not play a significant role on employment and wage structure shifts. Solving the general equilibrium model only matters for obtaining the wealth distribution.

**Figure 9. Job and Wage Polarization: Data vs Two-Sectors Model**
The modified model is promising in the sense that when it is reasonably well calibrated, it is able to capture both the employment and labor shifts in the labor markets. Unlike one-sector model, which generates a counterfactual decrease in the share of manual service jobs; two-sectors model is able to convert this counterfactual movement to an increase. Despite the correct directions of structural shifts, the increase generated in the share of manual jobs remains relatively modest compared to what is observed in data. Furthermore, the rise in the relative wage of abstract tasks are overstated in the model. Yet, the relative increase in wage of manual tasks is quite consistent with data. To sum it up, by switching to the two-sectors model, the model’s success to predict the relative wage trends remained the same, while the failure of the model in terms of generating the shifts in employment structure has been fixed to a great extent. There is still much room for potential improvement in results with a more sound calibration.

8 CONCLUSION

U.S. wage inequality has been rising in the past three decades. A recently growing literature has linked increased wage inequality with the recent job and wage polarization processes that have been observed in many developed countries in the same period. Meanwhile, both the supply of skilled labor and college premium has been increasing, implying that the rise in the demand for skill, which is evident from job polarization, has been dominating the effect of increase in supply.

Even though job and wage polarization might be explained by various factors such as globalization and offshoring, a great bulk of the literature focuses on routinization hypothesis as the main trigger of this employment and relative
wage shifts. In short, routinization hypothesis suggests that rapid decline in the price of computer capital replaces the routine tasks in production process, while inducing a higher demand for abstract and manual tasks. This results in an increase in employment shares and relative wages of both high and low skill occupations compared to middle skill ones.

In this paper, I tested the predictions of routinization hypothesis in an incomplete markets environment with heterogeneous agents. To do so, I combined various elements from standard models of both SBTC and job polarization literatures in a (Bewley 1977) type economy. Having incomplete markets enabled us to analyze the saving behavior of the agents under the presence of job polarization and establish the link between job polarization literature and wealth inequality. Furthermore, it allowed us to introduce a simple mechanism of education choice, thereby making the supply of skilled labor endogenous. That way, we were able to generate increase in supply and employment share of high skill labor following a rapid decline in computer price, unlike most of the standard models in job polarization, which usually lack any market imperfections and fail to generate the desired increase in the employment share of skilled labor.

To test the routinization hypothesis and analyze the implications of job and wage polarization on wealth accumulation, I first built and calibrated a heterogenous agents DSGE model with incomplete markets and job and wage polarization ingredients, that matched different aspects of U.S. wage and employment data for 1981. Later, I estimated the price of computer capital and solved the model again, while keeping the parameters the same as in 1981. Comparing two periods, which only differed in terms of price of computer capital, our model showed that routinization hypothesis alone accounts for a great part of increase in employment share of high skill occupations and their relative wages, as well as the increase in the supply of labor with a college equivalent degree. The model has also been successful in explaining the erosion of wealth in the middle of wealth distribution, while increasing share of asset holdings at the two extremes, thereby indicating the role that might have been played by job and wage polarization on the recently discussed phenomenon of middle class squeeze.

Despite its success in capturing most of the changes in labor markets and asset holdings, the model failed to generate a growth in the share of low skill occupations. On the contrary, it even resulted in a decline in the employment share of this group. As discussed extensively in earlier sections, this might be the result of choice of distribution of routine task efficiency and hence might
be overcome to some extent by disciplining this distribution from data. Fur-
thermore, another channel that would increase the demand for service (low
skill) occupations as computer prices decline should be introduced. This could
be achieved by introducing another sector, that uses only manual tasks and
produces a service that would be demanded more as there are more high skill
occupations with a relatively higher wages (Autor and Dorn 2013). Introducing
a multi-sector set up might also be crucial in explaining the differences in wage
inequality trends across sectors, which might be the topic of a future study.
References


